

# The Australian Curriculum

<b>Subjects</b>	Mathematics
<b>Year levels</b>	Year 10

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# **The Australian Curriculum**

## **Mathematics**

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# Mathematics - How the Subject works

## Rationale

Learning mathematics creates opportunities for and enriches the lives of all Australians. The Australian Curriculum: Mathematics provides students with essential mathematical skills and knowledge in *number and algebra, measurement and geometry, and statistics and probability*. It develops the numeracy capabilities that all students need in their personal, work and civic life, and provides the fundamentals on which mathematical specialties and professional applications of mathematics are built.



Mathematics has its own value and beauty and the Australian Curriculum: Mathematics aims to instil in students an appreciation of the elegance and power of mathematical reasoning. Mathematical ideas have evolved across all cultures over thousands of years, and are constantly developing. Digital technologies are facilitating this expansion of ideas and providing access to new tools for continuing mathematical exploration and invention. The curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, reasoning, and problem-solving skills. These proficiencies enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently.

The Australian Curriculum: Mathematics ensures that the links between the various components of mathematics, as well as the relationship between mathematics and other disciplines, are made clear. Mathematics is composed of multiple but interrelated and interdependent concepts and systems which students apply beyond the mathematics classroom. In science, for example, understanding sources of error and their impact on the confidence of conclusions is vital, as is the use of mathematical models in other disciplines. In geography, interpretation of data underpins the study of human populations and their physical environments; in history, students need to be able to imagine timelines and time frames to reconcile related events; and in English, deriving quantitative and spatial information is an important aspect of making meaning of texts.

The curriculum anticipates that schools will ensure all students benefit from access to the power of mathematical reasoning and learn to apply their mathematical understanding creatively and efficiently. The Mathematics curriculum provides students with carefully paced, in-depth study of critical skills and concepts. It encourages teachers to help students become self-motivated, confident learners through inquiry and active participation in challenging and engaging experiences.

## Aims

The Australian Curriculum: Mathematics aims to ensure that students:

- are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens
- develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in number and algebra,

- measurement and geometry, and statistics and probability
- recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.

## Key ideas

In Mathematics, the key ideas are the proficiency strands of understanding, fluency, problem-solving and reasoning. The proficiency strands describe the actions in which students can engage when learning and using the content. While not all proficiency strands apply to every content description, they indicate the breadth of mathematical actions that teachers can emphasise.

## Understanding

Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the ‘why’ and the ‘how’ of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

## Fluency

Students develop skills in choosing appropriate procedures; carrying out procedures flexibly, accurately, efficiently and appropriately; and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.

## Problem-solving

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

## Reasoning

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices.

[Click here for further information, illustrations of practice and student work samples portfolios for the Mathematics proficiencies](#)

## Structure

The Australian Curriculum: Mathematics is organised around the interaction of three content strands and four proficiency strands.

The content strands are *number and algebra*, *measurement and geometry*, and *statistics and probability*. They describe what is to be taught and learnt.

The proficiency strands are *understanding*, *fluency*, *problem-solving* and *reasoning*. They describe how content is explored or developed; that is, the thinking and doing of mathematics. The strands provide a meaningful basis for the development of concepts in the learning of mathematics and have been incorporated into the content descriptions of the three content strands. This approach has been adopted to ensure students' proficiency in mathematical skills develops throughout the curriculum and becomes increasingly sophisticated over the years of schooling.

# Content strands

## Number and algebra

Number and algebra are developed together, as each enriches the study of the other. Students apply number sense and strategies for counting and representing numbers. They explore the magnitude and properties of numbers. They apply a range of strategies for computation and understand the connections between operations. They recognise patterns and understand the concepts of variable and function. They build on their understanding of the number system to describe relationships and formulate generalisations. They recognise equivalence and solve equations and inequalities. They apply their number and algebra skills to conduct investigations, solve problems and communicate their reasoning.

## Measurement and geometry

Measurement and geometry are presented together to emphasise their relationship to each other, enhancing their practical relevance. Students develop an increasingly sophisticated understanding of size, shape, relative position and movement of two-dimensional figures in the plane and three-dimensional objects in space. They investigate properties and apply their understanding of them to define, compare and construct figures and objects. They learn to develop geometric arguments. They make meaningful measurements of quantities, choosing appropriate metric units of measurement. They build an understanding of the connections between units and calculate derived measures such as area, speed and density.

## Statistics and probability

Statistics and probability initially develop in parallel and the curriculum then progressively builds the links between them. Students recognise and analyse data and draw inferences. They represent, summarise and interpret data and undertake purposeful investigations involving the collection and interpretation of data. They assess likelihood and assign probabilities using experimental and theoretical approaches. They develop an increasingly sophisticated ability to critically evaluate chance and data concepts and make reasoned judgements and decisions, as well as building skills to critically evaluate statistical information and develop intuitions about data.

# Sub-strands

Content descriptions are grouped into sub-strands to illustrate the clarity and sequence of development of concepts through and across the year levels. They support the ability to see the connections across strands and the sequential development of concepts from Foundation to Year 10.

Table 1: Content strands and sub-strands in the Australian Curriculum: Mathematics (F–10)

Number and algebra	Measurement and geometry	Statistics and probability
Number and place value (F–8)	Using units of measurement (F–10)	Chance (1–10)
Fractions and decimals (1–6)	Shape (F–7)	Data representation and interpretation (F–10)
Real numbers (7–10)	Geometric reasoning (3–10)	N/A
Money and financial mathematics (1–10)	Location and transformation (F–7)	N/A
Patterns and algebra (F–10)	Pythagoras and trigonometry (9–10)	N/A
Linear and non-linear relationships (7–10)	N/A	N/A

# **The Australian Curriculum**

## **Mathematics**

### **Curriculum F-10**

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## Year 10 Level Description

The proficiency strands **understanding**, **fluency**, **problem-solving** and **reasoning** are an integral part of mathematics content across the three content strands: number and algebra, measurement and geometry, and statistics and probability. The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored or developed. They provide the language to build in the developmental aspects of the learning of mathematics. The achievement standards reflect the content and encompass the proficiencies.

At this year level:

- **understanding** includes applying the four operations to algebraic fractions, finding unknowns in formulas after substitution, making the connection between equations of relations and their graphs, comparing simple and compound interest in financial contexts and determining probabilities of two- and three-step experiments
  - **fluency** includes factorising and expanding algebraic expressions, using a range of strategies to solve equations and using calculations to investigate the shape of data sets
  - **problem-solving** includes calculating the surface area and volume of a diverse range of prisms to solve practical problems, finding unknown lengths and angles using applications of trigonometry, using algebraic and graphical techniques to find solutions to simultaneous equations and inequalities and investigating independence of events
  - **reasoning** includes formulating geometric proofs involving congruence and similarity, interpreting and evaluating media statements and interpreting and comparing data sets.
-

# Year 10 Content Descriptions

## Number and Algebra

### Money and financial mathematics

Connect the [compound interest](#) formula to repeated applications of [simple interest](#) using appropriate digital technologies ([ACMNA229 - Scootle ↗](#))



#### Elaborations

working with authentic information, data and interest rates to calculate compound interest and solve related problems



### Patterns and algebra

Factorise algebraic expressions by taking out a common algebraic factor ([ACMNA230 - Scootle ↗](#))



#### Elaborations

using the distributive law and the index laws to factorise algebraic expressions



understanding the relationship between factorisation and expansion



Simplify algebraic products and quotients using [index laws](#) ([ACMNA231 - Scootle ↗](#))



#### Elaborations

applying knowledge of index laws to algebraic terms, and simplifying algebraic expressions using both positive and negative integral indices



Apply the four operations to simple algebraic fractions with numerical denominators

([ACMNA232 - Scootle ↗](#))



## Elaborations

expressing the sum and difference of algebraic fractions with a common denominator



using the index laws to simplify products and quotients of algebraic fractions



Expand binomial products and factorise monic quadratic expressions using a variety of strategies  
(ACMNA233 - Scootle 



## Elaborations

exploring the method of completing the square to factorise quadratic expressions and solve quadratic equations



identifying and using common factors, including binomial expressions, to factorise algebraic expressions using the technique of grouping in pairs



using the identities for perfect squares and the difference of squares to factorise quadratic expressions



Substitute values into formulas to determine an unknown (ACMNA234 - Scootle 



## Elaborations

solving simple equations arising from formulas



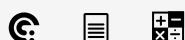
## Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas (ACMNA235 - Scootle 



## Elaborations

representing word problems with simple linear equations and solving them to answer questions



Solve linear inequalities and graph their solutions on a [number line \(ACMNA236 - Scootle\)](#)



Elaborations

representing word problems with simple linear inequalities and solving them to answer questions



Solve linear [simultaneous equations](#), using algebraic and graphical techniques, including using digital technology [\(ACMNA237 - Scootle\)](#)



Elaborations

associating the solution of simultaneous equations with the coordinates of the intersection of their corresponding graphs



Solve problems involving [parallel](#) and [perpendicular](#) lines [\(ACMNA238 - Scootle\)](#)



Elaborations

solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel



solving problems using the fact that the product of the gradients of perpendicular lines is  $-1$  and conversely that if the product of the gradients of two lines is  $-1$  then they are perpendicular



Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate [\(ACMNA239 - Scootle\)](#)



Elaborations

sketching graphs of parabolas, and circles



applying translations, reflections and stretches to parabolas and circles



sketching the graphs of exponential functions using transformations



Solve linear equations involving simple algebraic fractions (ACMNA240 - Scootle 



Elaborations

solving a wide range of linear equations, including those involving one or two simple algebraic fractions, and checking solutions by substitution



representing word problems, including those involving fractions, as equations and solving them to answer the question



Solve simple quadratic equations using a range of strategies (ACMNA241 - Scootle 



Elaborations

using a variety of techniques to solve quadratic equations, including grouping, completing the square, the quadratic formula and choosing two integers with the required product and sum



## Measurement and Geometry

### Using units of measurement

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242 - Scootle 



Elaborations

investigating and determining the volumes and surface areas of composite solids by considering the individual solids from which they are constructed



### Geometric reasoning

Formulate proofs involving congruent triangles and angle properties (ACMMG243 - Scootle 



#### Elaborations

applying an understanding of relationships to deduce properties of geometric figures (for example the base angles of an isosceles triangle are equal)



Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244 - Scootle 



#### Elaborations

distinguishing between a practical demonstration and a proof (for example demonstrating triangles are congruent by placing them on top of each other, as compared to using congruence tests to establish that triangles are congruent)



performing a sequence of steps to determine an unknown angle giving a justification in moving from one step to the next.



communicating a proof using a sequence of logically connected statements



## Pythagoras and trigonometry

Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245 - Scootle 



#### Elaborations

applying Pythagoras' Theorem and trigonometry to problems in surveying and design



## Statistics and Probability

### Chance

Describe the results of two- and three-step chance experiments, both with and without replacements,

assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence ([ACMSP246 - Scootle ↗](#))



#### Elaborations

recognising that an event can be dependent on another event and that this will affect the way its probability is calculated



Use the language of 'if ....then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language ([ACMSP247 - Scootle ↗](#))



#### Elaborations

using two-way tables and Venn diagrams to understand conditional statements



using arrays and tree diagrams to determine probabilities



## Data representation and interpretation

Determine quartiles and [interquartile range \(ACMSP248 - Scootle ↗\)](#)

#### Elaborations

finding the five-number summary (minimum and maximum values, median and upper and lower quartiles) and using its graphical representation, the box plot, as tools for both numerically and visually comparing the centre and spread of data sets



Construct and interpret box plots and use them to compare [data sets \(ACMSP249 - Scootle ↗\)](#)



#### Elaborations

understanding that box plots are an efficient and common way of representing and summarising data and can facilitate comparisons between data sets



using parallel box plots to compare data about the age distribution of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole



Compare shapes of box plots to corresponding histograms and dot plots (ACMSP250 - Scootle



#### Elaborations

investigating data in different ways to make comparisons and draw conclusions



Use scatter plots to investigate and comment on relationships between two numerical variables (ACMSP251 - Scootle



#### Elaborations

using authentic data to construct scatter plots, make comparisons and draw conclusions



Investigate and describe bivariate numerical data where the independent variable is time (ACMSP252 - Scootle



#### Elaborations

investigating biodiversity changes in Australia since European occupation



constructing and interpreting data displays representing bivariate data over time



Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (ACMSP253 - Scootle



#### Elaborations

investigating the use of statistics in reports regarding the growth of Australia's trade with other countries of the Asia region



evaluating statistical reports comparing the life expectancy of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole



## Year 10 Achievement Standards

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

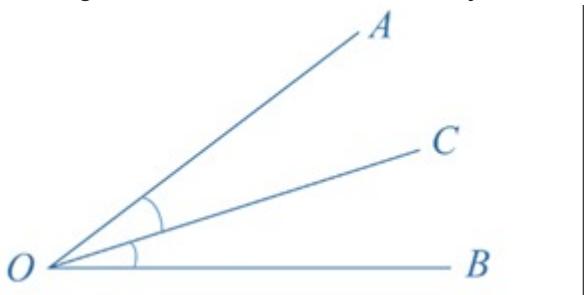
## Glossary

### acute angle

An *acute angle* is bigger than  $0^\circ$  but smaller than  $90^\circ$ .

### adjacent angles

Adjacent angles share a common ray and a common vertex, and lie on opposite sides of the common ray. In the diagram,  $\angle AOC$  and  $\angle COB$  are adjacent angles.



### algebraic expression

An algebraic expression is formed by combining numbers and algebraic terms using arithmetic operations (addition, subtraction, multiplication, division, and exponentiation). The expression must be unambiguous. For example,  $(a^2+3ab-2b^2)$  is an algebraic expression, but  $(2x+\text{div}3)$  is not one because it is ambiguous.

### algebraic fraction

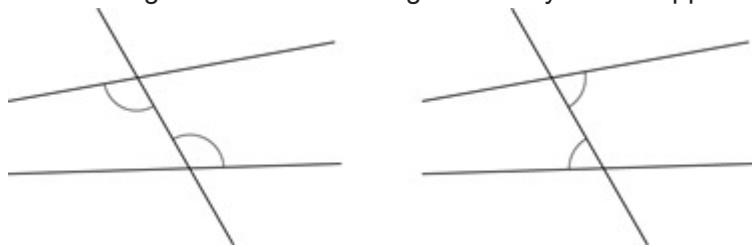
An *algebraic fraction* is a fraction in which both, *numerator* and *denominator*, are *algebraic expressions*.

### algebraic term

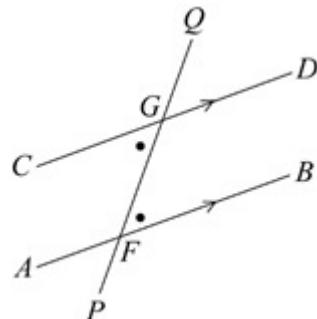
An algebraic term forms a part of an algebraic expression. For example,  $(1;2,\text{ }3x)$ , and  $(5x^2)$  are terms of the algebraic expression  $(2+3x-5x^2)$ . Terms are separated by + or – signs.

## alternate angles

Alternate angles are formed when two lines are crossed by another line (the transversal). The alternate angles are on opposite sides of the transversal, but inside the two lines. In each diagram below, the two marked angles are alternate angles as they are on opposite sides of the transversal.



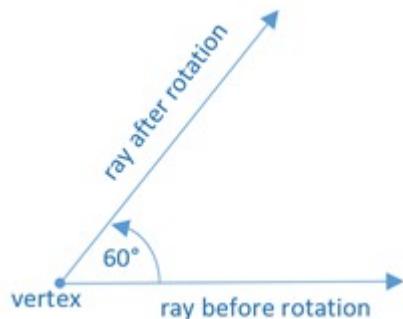
If the lines AB and CD are parallel, then each pair of alternate angles are equal.



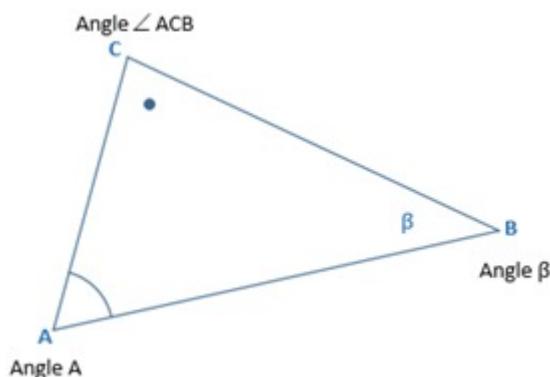
Conversely, if a pair of alternate angles are equal, then the lines are parallel. Line segment CD is parallel to line segment AB, because  $\angle CGF$  equals  $\angle GFB$ .

## angle

An angle is the figure formed by the rotation of a ray about a point, called the vertex of the angle. The size of an angle is usually measured in degrees ( $^{\circ}$ ).

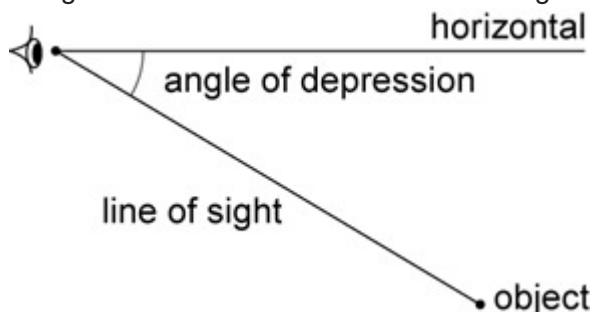


There are different ways of depicting and naming angles. Angles may be depicted using a symbol such as an arc, a dot or a letter (often from the Greek alphabet). Angles are named using different conventions, such as the angle symbol  $\angle$  followed by three letters denoting points, where the middle letter is the vertex, or using just the label of the vertex.



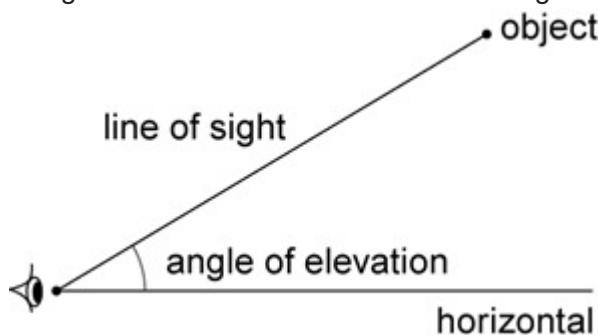
## angle of depression

When an observer looks at an object that is lower than the eye of the observer, the angle between the line of sight and the horizontal is called the angle of depression.



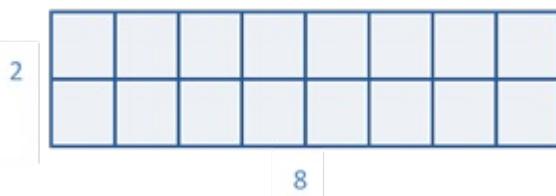
## angle of elevation

When an observer looks at an object that is higher than the eye of the observer, the angle between the line of sight and the horizontal is called the angle of elevation.



## area

Area is a measure of how many units are required to cover a surface. The units are usually standard units, such as square centimetres or square metres. The area of a rectangle can be found by multiplying the size of its length by the size of its width. For example, the area of the rectangle below is given by  $8 \times 2 = 16$  units.



## array

An array is an ordered collection of objects or numbers. Rectangular arrays are commonly used in primary mathematics. For example, the two arrays of dots shown below are two different representations of the number 24.



## associative

Operations are associative if the order in which operations take place does not affect the result. For example, addition of numbers is associative, since the order in which they are added does not change their sum. The corresponding associative law is:  $(a+b)+c=a+(b+c)$  for all numbers **a, b and c**. Multiplication is also associative, as the product of the numbers does not vary with the order of their multiplication. The corresponding associative law is:  $(ab)c=a(bc)$  for all numbers **a, b and c**. Subtraction and division are not associative, as the order of operations changes the value of the expression; for example:  $(7-4)-3 \neq 7-(4-3)$  and  $(12 \div 6) \div 2 \neq 12 \div (6 \div 2)$ .

## average

An **average** is a number expressing a central or typical value in a *set of data*. While it usually refers to the arithmetic *mean*, that is, the *sum* of a set of numbers divided by the number of numbers in the set, it may also refer to other *measures of central tendency*.

## axes

(plural) See *axis*.

## axis

(singular) An axis is one of the horizontal or vertical lines that make up the Cartesian plane. The horizontal line is called the  $\backslash(x\backslash)$  axis, and the vertical line is called the  $\backslash(y\backslash)$  axis. The  $\backslash(x\backslash)$  and  $\backslash(y\backslash)$  axes intersect at point O, called the origin, which defines the centre of the coordinate system.

## back-to-back stem and leaf plot

A back-to-back stem and leaf plot is a method for comparing two data distributions by attaching two sets of ‘leaves’ to the same ‘stem’; for example, the stem-and-leaf plot below displays the distribution of pulse rates of 19 students before and after gentle exercise.

**pulse rate**

<b>before</b>	<b>after</b>
9 8 8 8	6
8 6 6 4 1 1 0	7
8 8 6 2	8 6 7 8 8
6 0	9 0 2 2 4 5 8 9 9
4	10 0 4 4
0	11 8
	12 4 4
	13
14	6

In this plot, the stem unit is ‘10’ and the leaf unit is ‘1’. Thus, the third row in the plot, 8 8 6 2 | 8 | 6 7 8 8, displays pulse rates of 88, 88, 86, 82 before exercise and 86, 87, 88, 88 after exercise.

## bimodal data

*Bimodal data* has two modes.

## binomial

A binomial is an algebraic expression containing two distinct algebraic terms. For example,  $(2x+a)$  and  $(2x+8)$  are binomial expressions but  $(3x+2x)$  is not, as it can be simplified to  $(5x)$ .

## bivariate data

*Bivariate data* relates to two *variables*. For example, the arm spans and heights of 16-year-olds or the sex of primary school students and their attitudes toward playing sports.

## bivariate numerical data

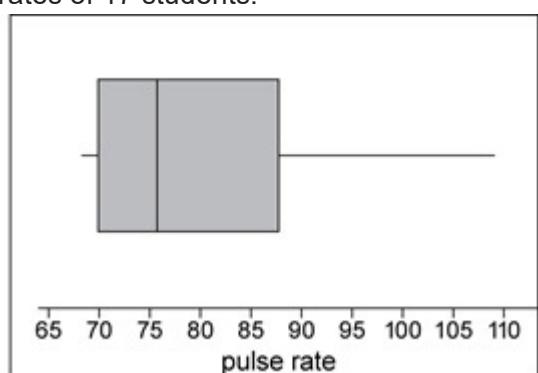
*Bivariate numerical data* relates to two *numerical variables*. For example, height and weight.

## box-and-whisker plot

A box-and-whisker plot is a graphical display of a five-number summary.

In a box-and-whisker plot, the ‘box’ covers the interquartile range (IQR), the middle 50% of scores, with ‘whiskers’ reaching out from each end of the box to indicate maximum and minimum values in the data set. A vertical line in the box is used to indicate the location of the median.

The box-and-whisker plot below has been constructed from the five-number summary of the resting pulse rates of 17 students.



## box plot

The term *box plot* is a synonym for a *box-and-whisker plot*.

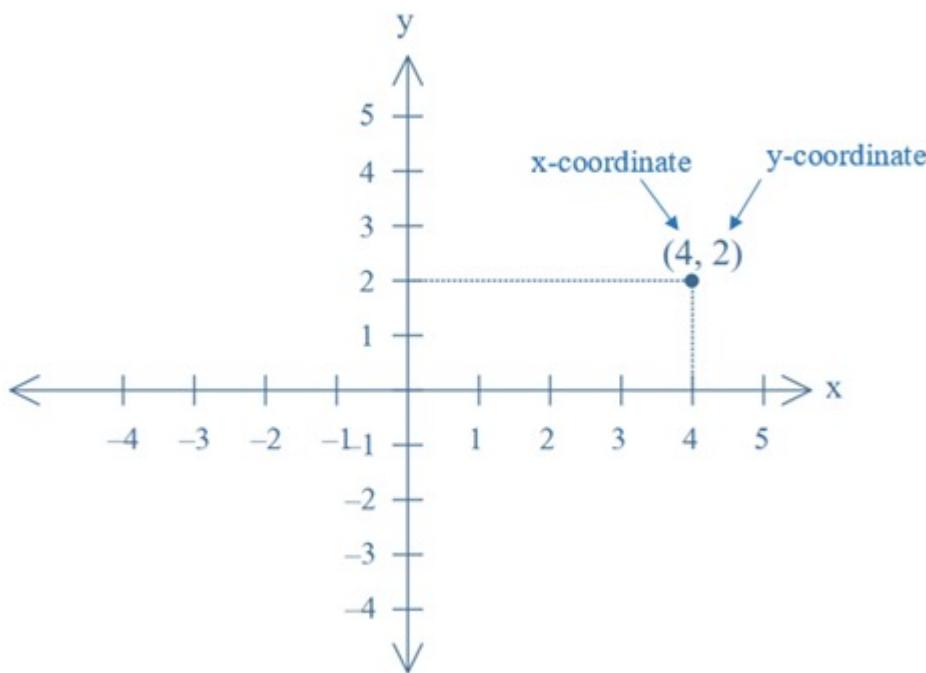
## capacity

In the given context, *capacity* is a term that describes how much a container will hold. It is used in reference to the volume of fluids or gases and is measured in units such as litres or millilitres.

## Cartesian plane

The Cartesian plane or Cartesian coordinate system is a system that describes the exact location of any point in a plane using an ordered pair of numbers, called coordinates. It is defined by the intersection of a horizontal and vertical number line at a point called the origin. The coordinates of the origin are (0, 0).

The Cartesian plane is divided into four quadrants by these perpendicular axes called the x-axis (horizontal line) and the y-axis (vertical line). The axes can be used to identify any point in the plane using a pair of coordinates, as shown in the diagram below.



## categorical variable

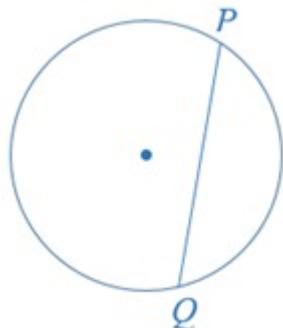
A *categorical variable* is a *variable* whose values are categories. For example, blood group is a categorical variable; its common values are: A, B, AB or O.

## census

A *census* is a survey of a whole *population*.

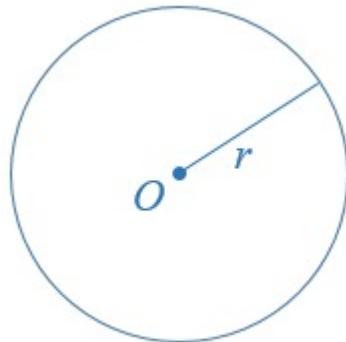
## chord

A chord in a circle is a line segment joining any two points on the circle. Chord PQ, illustrated below, joins points P and Q.



## circle

A circle, with centre O and radius r, is the set of all points on a plane whose distance from O is r.



## circumference

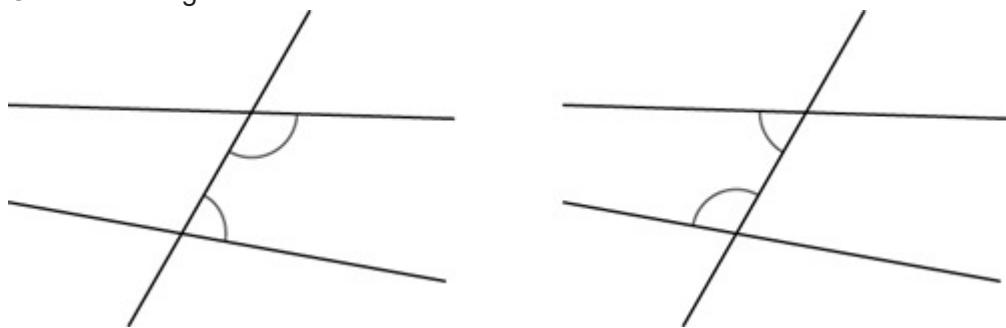
Circumference refers to the boundary of a circle. The length of the circumference  $c$  is given by  $(c=\pi d)$ , where  $d$  is the diameter. Alternatively, it is given by  $(c=2\pi r)$ , where  $r$  is the radius.

## classification of angles

Angles are classified according to their size. See *acute angle*, *obtuse angle*, *reflex angle*, *right angle*, *straight angle* and *revolution*.

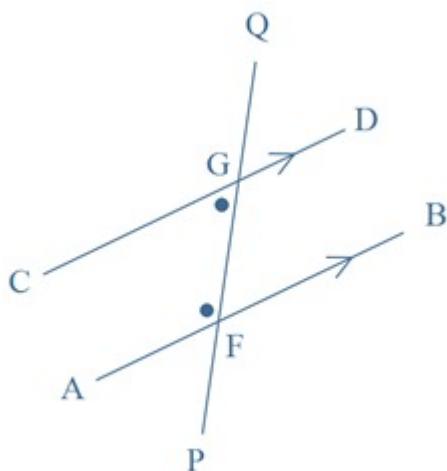
## co-interior angles

Co-interior angles lie between two lines and on the same side of a transversal.



In each diagram the two marked angles are called co-interior angles.

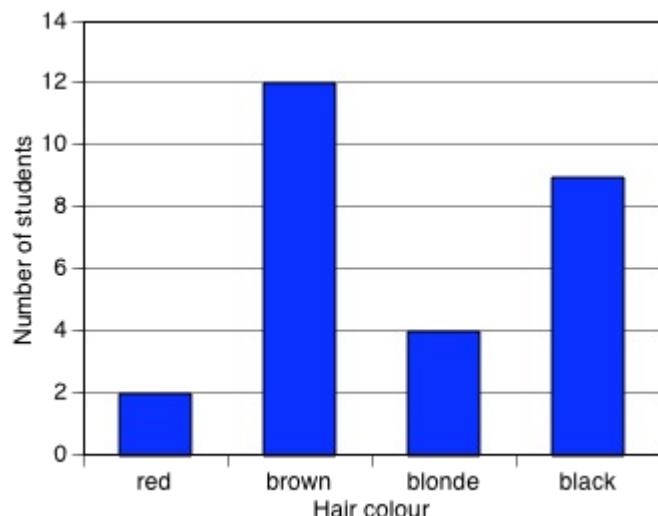
If the two lines are parallel, then co-interior angles add to give  $180^\circ$  and so are supplementary. In the diagram below the angles  $\angle CGF$  and  $\angle AFG$  are supplementary.



Conversely, if a pair of angles are supplementary, then the lines are parallel. Line segment CD is parallel to line segment AB, because  $\angle CGF + \angle AFG = 180^\circ$ .

## column graph

A column graph is a graph used in statistics for organising and displaying categorical data. It consists of a series of equal-width rectangular columns, one for each category. Each column has a height equal to the frequency of the category. This is shown in the example below which displays the hair colours of 27 students.



Column graphs are frequently called bar graphs or bar charts. In a bar graph or chart, the bars can be either vertical or horizontal.

## common factor

A common factor (or common divisor) of a set of numbers or algebraic expressions is a factor of each element of that set. For example, 6 is a common factor of 24, 54 and 66, since  $(24=6\times 4)$ ,  $(54=6\times 9)$ , and  $(66=6\times 11)$ . Similarly,  $(x+1)$  is a common factor of  $(x^2-1)$  and  $(x^2+5x+4)$ , since  $(x^2-1)=(x+1)(x-1)$  and  $(x^2+5x+4)=(x+1)(x+4)$ .

## commutative operations

Operations are commutative if the order in which terms are given does not affect the result.

The commutative law for addition is:  $(a+b=b+a)$  for all numbers a and b.

For example,  $3+5=5+3$ .

The commutative law for multiplication is:  $(ab=ba)$  for all numbers a and b.

For example,  $4\times 7=7\times 4$ .

Subtraction and division are not commutative because  $5-3\neq 3-5$  and  $12\div 4\neq 4\div 12$ .

## complementary angles

Two angles that add to  $90^\circ$  are called *complementary*; for example,  $23^\circ$  and  $67^\circ$  are *complementary angles*.

## complementary events

Events A and B are complementary events if A and B are mutually exclusive (have no overlap) and  $(Pr(A)+Pr(B)=1)$ , where the symbol  $(Pr(A))$  denotes the probability of event  $(A)$  occurring.

## composite number

A *composite number* is a *natural number* that has a *factor* other than 1 and itself.

## compound interest

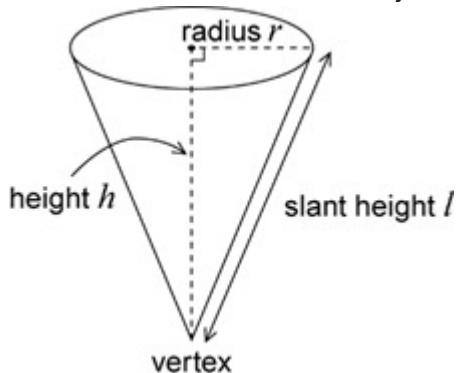
The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment. For example, if the principal  $\$P$  earns compound interest at the rate of  $r\%$  per period, then after  $n$  periods the principal plus interest is  $\$P(1+r)^n$ .

## computation

*Computation* is mathematical calculation.

## cone

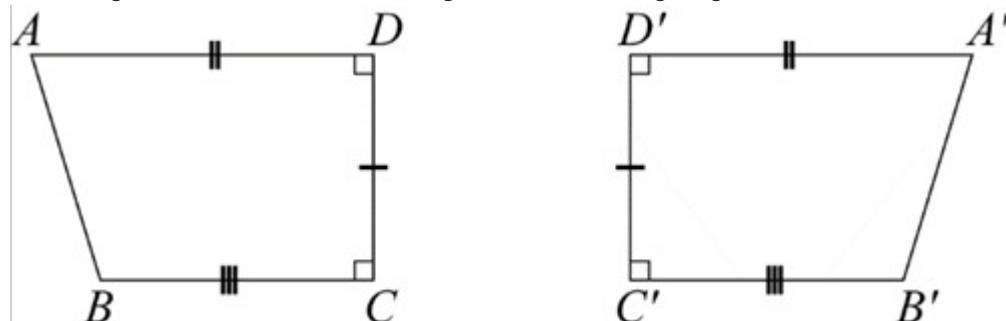
A cone is a solid that is formed by taking a circle, called the base, and a point, called the vertex, which lies above or below the circle, and joining the vertex to each point on the circle.



## congruence

Two plane shapes are congruent if they are identical in size and shape and one can be moved or reflected so that it fits exactly on top of the other figure.

Matching sides have the same length, and matching angles have the same size.



The four standard congruence tests for triangles

Two triangles are congruent if:

SSS: the three sides of one triangle are respectively equal to the three sides of the other triangle, or

SAS: two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the other triangle, or

AAS: two angles and one side of one triangle are respectively equal to two angles and the matching side of the other triangle, or

RHS: the hypotenuse and one side of one right-angled triangle are respectively equal to the hypotenuse and one side of the other right-angled triangle.

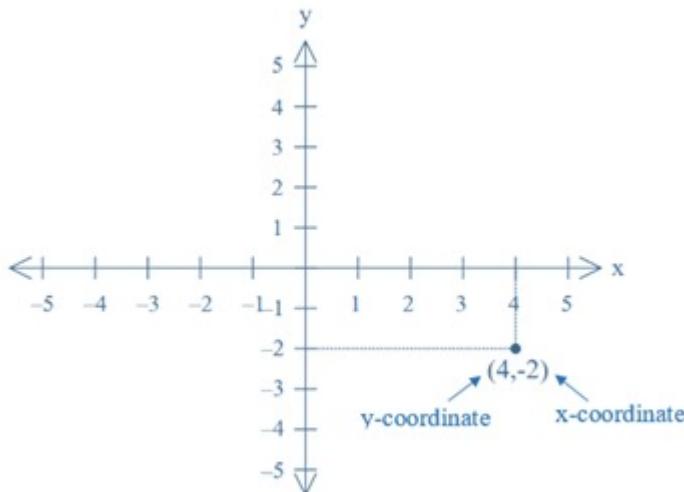
## continuous numerical data

*Continuous numerical data* includes any value that lies within an *interval*. In practice, the values taken are subject to the accuracy of the measurement instrument used to obtain these values. Height, reaction time to a stimulus and systolic blood pressure are all types of continuous numerical data that can be collected.

## coordinate

A coordinate is one value of an ordered pair that describes the location of a point along an axis in the Cartesian plane. By definition, the first number ( $x$ -coordinate) of the ordered pair denotes the horizontal distance, the second number ( $y$ -coordinate) gives the vertical distance from the centre (origin) of the coordinate system. Positive  $x$  coordinates indicate that the point is located to the right (East), negative to the left (West) of the origin. Positive  $y$  coordinates indicate a location above (North of), negative below (South of) the origin. The origin has the coordinates (0,0).

For instance, in the ordered pair (4, -2) the number 4 denotes the  $x$  coordinate of a point situated at a horizontal distance of 4 units to the origin. The number -2 denotes the  $y$  coordinate of the same point indicating a vertical distance of 2 units below the origin.

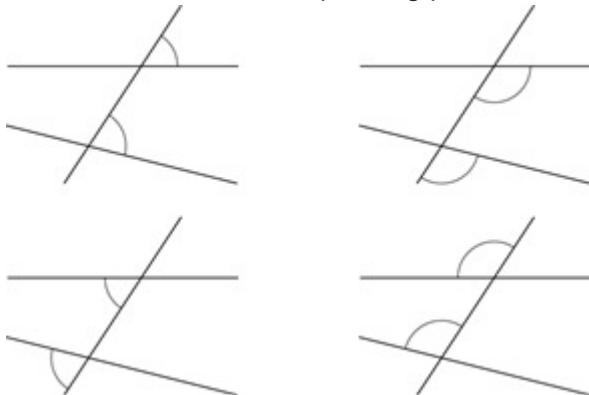


## coordinate system

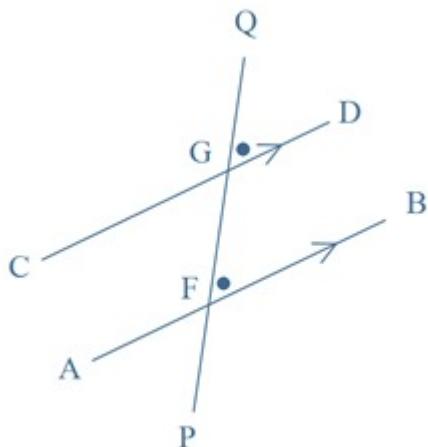
see Cartesian plane.

## corresponding angles

Corresponding angles are formed when two lines are crossed by another line (the transversal). In each diagram the two marked angles are called corresponding angles because they are on the same side of the transversal and in corresponding positions in relation to the lines.



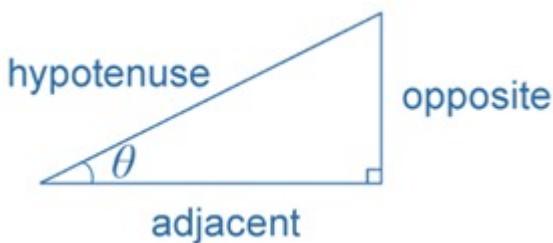
If the lines are parallel, then each pair of corresponding angles is equal (as are the angles  $\angle QGD$  and  $\angle GFB$  in the diagram shown below).



Conversely, if a pair of corresponding angles is equal, then the lines are parallel.

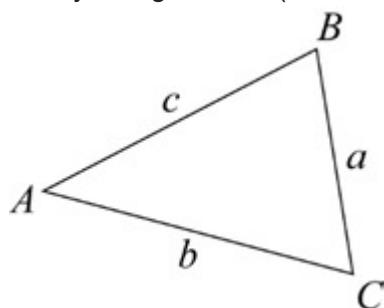
## cosine

In any right-angled triangle,  $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ , where  $0 < \theta < 90^\circ$ .



## cosine rule

In any triangle ABC,  $(c^2 = a^2 + b^2 - 2ab \cos C)$



## counting numbers

*Counting numbers* are the *positive integers*, that is, the numbers 1, 2, 3, ... .

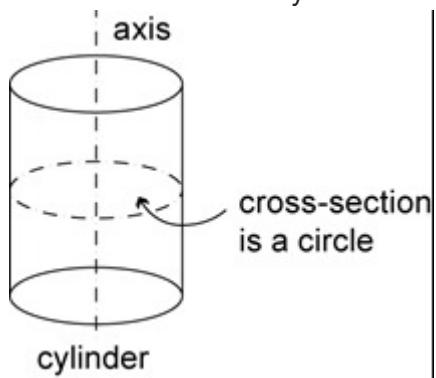
Sometimes it is taken to mean the non-negative *integers*, which include zero.

## counting on

*Counting on* is a strategy for solving simple addition problems. For example, a student can add 6 and 4 by counting on from 6, saying '7, 8, 9, 10'. If students are asked how many more objects need to be added to a collection of 8 to give a total of 13, they can count '9, 10, 11, 12, 13' to find the answer 5.

## cylinder

A cylinder is a solid that has parallel circular discs of equal radius at the ends, and whose horizontal cross-section is a circle with the same radius. The centres of these circular cross-sections lie on a straight line, called the axis of the cylinder.



## data

*Data* is a general term for information (observations and/or measurements) collected during any type of systematic investigation.

## data display

A *data display* is a visual format for organising and summarising *data*. Examples include *box plots*, *column graphs*, *frequency tables*, *scatter plots*, and *stem plots*.

## decimal

A decimal is a numeral in the decimal number system, which is the place-value system most commonly used for representing real numbers. In this system numbers are expressed as sequences of Arabic numerals 0 to 9, in which each successive digit to the left or right of the decimal point indicates a multiple of successive powers of 10; for example, the number represented by the decimal 123.45 is the sum

$$(1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2})$$

$$(=1 \times 100 + 2 \times 10 + 3 \times 1 + 4 \times \frac{1}{10} + 5 \times \frac{1}{100})$$

The digits after the decimal point can be terminating or non-terminating. A terminating decimal is a decimal that contains a finite number of digits, as shown in the example above. A decimal is non-terminating, if it has an infinite number of digits after the decimal point. Non-terminating decimals may be recurring, that is, contain a pattern of digits that repeats indefinitely after a certain number of places. For example, the fraction  $(\frac{1}{3})$ , written in the decimal number system, results in an infinite sequence of 3s after the decimal point. This can be represented by a dot above the recurring decimal.

$$(\frac{1}{3}=0.\overline{3})$$

Similarly, the fraction  $(\frac{1}{7})$  results in a recurring group of digits, which is represented by a bar above the whole group of repeating digits

$$(\frac{1}{7}=0.\overline{142857})$$

Non-terminating decimals may also be non-recurring, that is the digits after the decimal point never repeat in a pattern. This is the case for irrational number, such as pi, e, or  $(\sqrt{2})$ . For example,

$$(\pi=3.1415926535897932384626433832795028841971693993751058209749\dots)$$

Irrational numbers can only be approximated in the decimal number system.

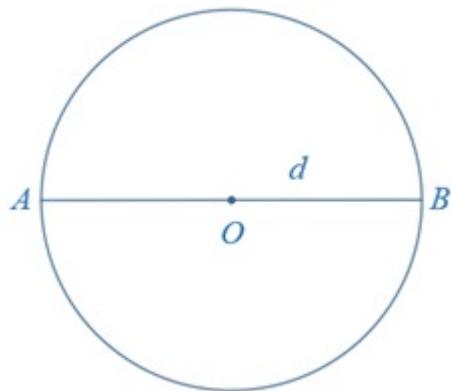
## denominator

In any fraction in the form  $(\frac{a}{b})$ , b is the denominator. It represents the number of equal parts into which the whole has been divided. For example, in the diagram below, a rectangle has been divided into 5 equal parts. Each of those parts is one fifth of the whole and corresponds to the unit fraction  $(\frac{1}{5})$ .



## diameter

A diameter is a chord that passes through the centre of a circle. The word diameter is also used to refer to the length of the diameter. The diameter  $d$  of the circle below is represented by line segment AB.



## difference

A **difference** is the result of subtracting one number or algebraic quantity from another. For example, the difference between 8 and 6 is 2, written as  $8-6=2$ .

## distributive

Multiplication of numbers is said to be ‘distributive over addition’, because the product of one number with the sum of two others equals the sum of the products of the first number with each of the others. For example, the product of 3 with  $(4+5)$  gives the same result as the sum of  $3 \times 4$  and  $3 \times 5$ :

$$3 \times (4+5) = 3 \times 9 = 27 \text{ and } 3 \times 4 + 3 \times 5 = 12 + 15 = 27$$

This distributive law is expressed algebraically as follows:

$$a(b+c) = ab+ac, \text{ for all numbers } a, b \text{ and } c.$$

## divisible

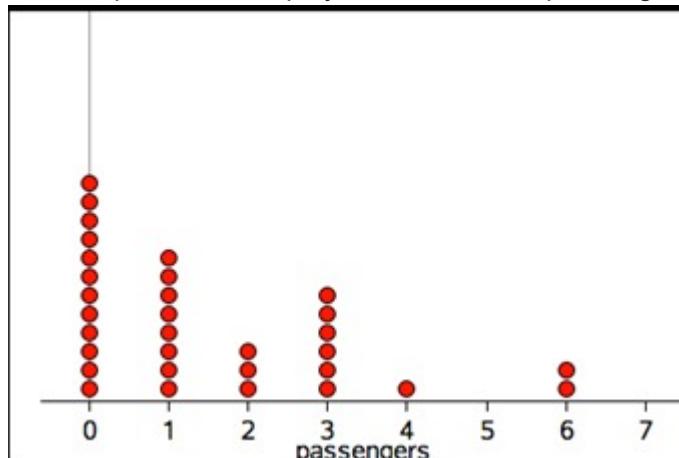
In general, a number or algebraic expression  $\langle x \rangle$  is divisible by another  $\langle y \rangle$ , if there exists a number or algebraic expression  $\langle q \rangle$  of a specified type for which  $\langle x=yq \rangle$ .

A natural number  $\langle m \rangle$  is divisible by a natural number  $\langle n \rangle$  if there is a natural number  $\langle q \rangle$  such that  $\langle m=nq \rangle$ ; for example, 12 is divisible by 4 because  $12=3 \times 4$ .

## dot plot

A dot plot is a graph used in statistics for organising and displaying categorical data or discrete numerical data.

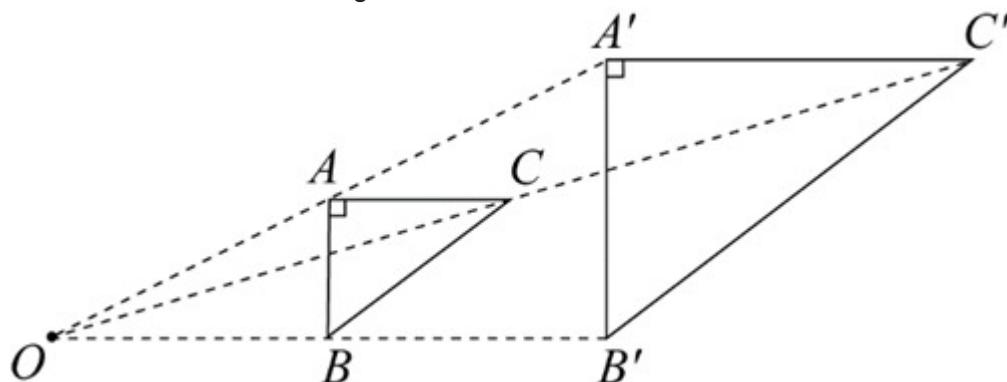
The dot plot below displays the number of passengers observed in 32 cars stopped at a traffic light.



## enlargement

An enlargement is a scaled up (or down) version of a figure so that the new figure is in proportion to the original figure. The relative positions of points are unchanged and the two figures are similar.

In the diagram below triangle  $A'B'C'$  is the image of triangle  $ABC$  under the enlargement with enlargement factor 2 and centre of enlargement  $O$ .



## equally likely outcomes

Equally likely outcomes have the same probability of occurring. For example, in tossing a fair coin, the outcome ‘head’ and the outcome ‘tail’ are equally likely. In this situation,  
 $\Pr(\text{head}) = \Pr(\text{tail}) = 0.5$ .

## equation

An equation is a statement that asserts that two mathematical expressions are equal in value. An equation must include an equal sign.

Examples of equations are  $(3+14=6+11)$  or  $(2x+5=21)$ .

## equivalent fractions

Equivalent fractions are alternative ways of writing the same fraction; for example,  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$  etc. Two fractions  $\frac{a}{b}$  and  $\frac{c}{d}$  are equivalent, if they are equal in value, that is, if  $(ad=bc)$ .

## estimate

To estimate is to judge the value, number, or quantity of a calculation roughly.

In statistical terms, an estimate is information about a population extrapolated from a sample of the population; for example, the mean number of decayed teeth in a randomly selected group of eight-year-old children is an estimate of the mean number of decayed teeth in eight-year-old children in Australia.

## event

An *event* is a subset of the *sample space* for a *random experiment*; for example, the set of outcomes from tossing two coins is {HH, HT, TH, TT}, where H represents a ‘head’ and T a ‘tail’.

## even number

An *even number* is an *integer* that is *divisible* by 2. The even numbers are ..., -4, -2, 0, 2, 4, ... .

## exponential function

An exponential function is a function where the independent variable is in the exponent (or index), that is, in the simplest form,  $f(x)=a^x$ , where  $a$  is a positive real number not equal to zero.

## expression

An expression refers to two or more numbers or variables connected by operations. For example,  $(17-9)$ ,  $8\times(2+3)$ ,  $(2a+3b)^2$  are all expressions. Expressions do not include an equal sign.

## factor

Numbers or algebraic expressions are factors (or divisors) of another number if they multiply to give that number. For example, 3 and 4 are factors of 12 as  $3\times4=12$ . This can be written algebraically as  $(x)$  and  $(y)$  are factors of  $(m)$ , if  $(m=xy)$ .

For polynomial expressions the same rule applies. For example,  $(x-4)$  and  $(x-2)$  are factors of the quadratic expression  $(x^2-6x+8)$  because  $((x-4)(x-2)=x^2-6x+8)$ .

## factor and remainder theorem

According to the factor theorem, if  $(p(x))$  is a polynomial and  $(p(a)=0)$  for some number  $(a)$ , then  $(x-a)$  is a factor of  $(p(x))$ . Conversely, if  $(p(x))$  is divisible by  $(x-a)$  then  $(p(a)=0)$ .

This follows from the more general remainder theorem, which states that the remainder of the division of a polynomial  $(p(x))$  by a linear polynomial  $(x-a)$  is equal to  $(p(a))$ . This relationship is often stated in the form  $(px=q(x)(x-a)+p(a))$ , where  $(q(x))$  is another polynomial, usually referred to as the quotient. It follows that, if  $(p(a)=0)$ , the remainder is  $(0)$  and  $(p(x))$  is divisible by  $(x-a)$ .

The factor theorem can be used to obtain factors of a polynomial; for example, if  $(p(x)=x^3-3x^2+5x-6)$ , then it is easy to check that  $(p(2)=2^3-3\times2^2+5\times2-6=0)$ . So by the factor theorem  $(x-2)$  is a factor of  $(x^3-3x^2+5x-6)$ .

## factorise

To factorise a number or algebraic expression is to express it as a product; for example, 15 is factorised when expressed as a product:  $15=3\times5$ .

$(x^2-3x+2)$  is factorised when written as a product:  $(x^2-3x+2=(x-1)(x-2))$

## five-number summary

A *five-number summary* is a method of summarising a data set using five *statistics*: the minimum value, the lower *quartile*, the *median*, the upper *quartile* and the maximum value. *Box plots* are a useful method of graphically depicting five-number summaries.

## fraction

The *fraction*  $(\frac{ab})$  (written alternatively as  $a/b$ ), where  $a$  and  $b$  are integers unequal to zero. For example,  $(\frac{3}{5})$  refers to 3 of 5 equal parts of the whole.

In the fraction  $(\frac{ab})$  the number  $a$  is the *numerator* and the number  $b$  is the *denominator*.

## frequency

Frequency, or observed frequency, is the number of times that a particular value occurs in a data set. For grouped data, it is the number of observations that lie in that group or class interval.

An expected frequency is the number of times that a particular event is expected to occur when a chance experiment is repeated a number of times. If the experiment is repeated  $n$  times, and on each of those times the probability that the event occurs is  $p$ , then the expected frequency of the event is  $np$ .

For example, suppose that a fair coin is tossed 5 times and the number of heads showing recorded. Then the expected frequency of 'heads' is  $5/2$ . This example shows that the expected frequency is not necessarily an observed frequency, which in this case is any one of the numbers 0, 1, 2, 3, 4 or 5.

The relative frequency is given by the ratio  $(\frac{fn})$ , where  $f$  is the frequency of occurrence of a particular data value or group of data values in a data set and  $n$  is the number of data values in the data set.

## frequency distribution

A *frequency distribution* is the division of a set of observations into a number of classes, together with a listing of the number of observations (the *frequency*) in that class. Frequency distributions can be displayed in the form of a *frequency table*, a *two-way-table* or in graphical form.

## frequency table

A frequency table lists the frequency (number of occurrences) of observations in different ranges, called class intervals.

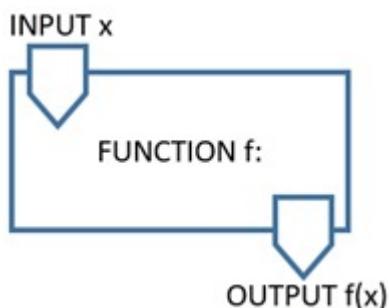
The frequency distribution of the heights (in cm) of a sample of 46 people is displayed in the form of a frequency table below.

Height (cm)	
Class interval	Frequency
155-160	3
160-165	2
165-170	9
170-175	7
175-180	10
180-185	5
185-190	5
190-195	5

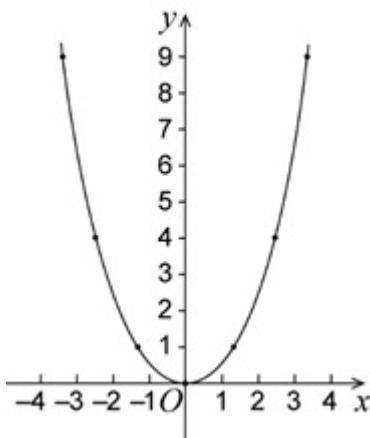
The information in a frequency table can also be displayed graphically in the form of a histogram or using a column graph.

## function

A function  $f$  assigns to each element of a set of input values (called the domain) precisely one element of a set of output values (called the range). In mathematical modelling, the independent variable is usually chosen as the input values for the function. The output values then represent the dependent variable.



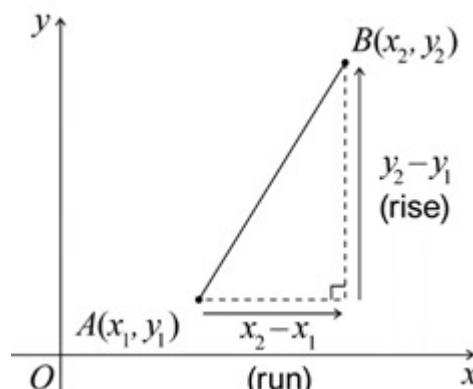
Functions are usually defined by a formula for  $f(x)$  in terms of  $x$ ; for example, the formula  $f(x)=x^2$  defines the ‘squaring function’ that maps each real number  $x$  to its square  $x^2$ . The graph of this function is shown below.



## gradient

The gradient of a line is sometimes also called a slope and is a measure of how steeply a line is rising or falling.

If  $(A(x_1, y_1))$  and  $(B(x_2, y_2))$  are points on the plane, the gradient of the line segment AB is given by  $\text{gradient}(AB) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ , provided that  $(x_2 - x_1 \neq 0)$

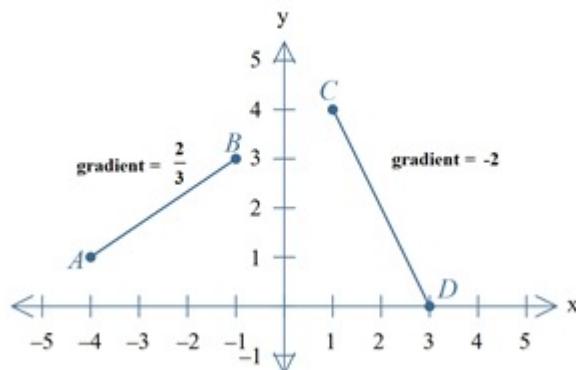


The gradient of a line is the gradient of any line segment within the line.

Gradients can be positive or negative, indicating whether the line is increasing or decreasing from left to right. The graph below shows two examples:

$$\text{gradient}(AB) = \frac{3-1}{-1-(-4)} = \frac{2}{3}$$

$$\text{gradient}(CD) = \frac{0-4}{3-1} = \frac{-4}{2} = -2$$



## grid reference

A grid reference identifies a region on a map. Coordinates and gridlines are used to refer to specific features or locations. For example, in the map below, the school is located at the grid reference C4.

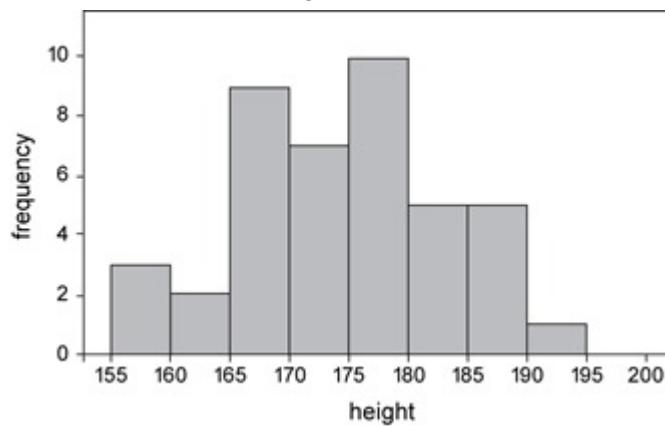


## histogram

A histogram is a statistical graph for displaying the frequency distribution of continuous data.

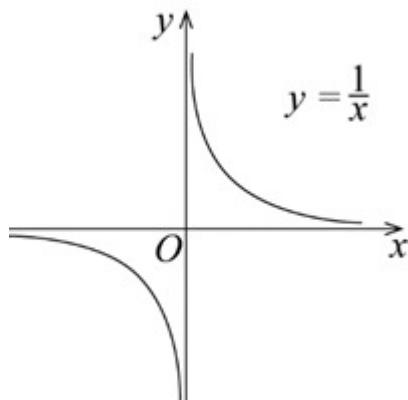
A histogram is a graphical representation of the information contained in a frequency table. In a histogram, class frequencies are represented by the areas of rectangles centred on each class interval. The class frequency is proportional to the rectangle's height when the class intervals are all of equal width.

The histogram below displays the frequency distribution of the heights (in cm) of a sample of 42 people with class intervals of width 5 cm.



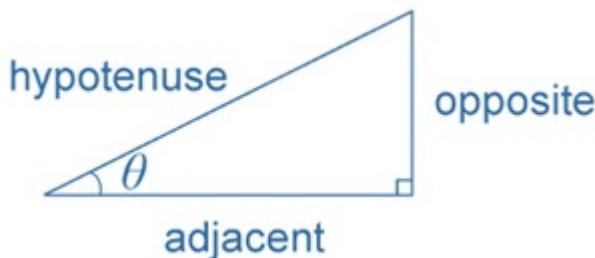
## hyperbola

A hyperbola is the graph of a curve in two parts separated by straight lines called asymptotes. The simplest example is the graph of  $y = \frac{1}{x}$ , called a rectangular hyperbola.



## hypotenuse

The hypotenuse is the side opposite the right angle. It is also always the longest side in a right-angled triangle.



## image

In geometry, an *image* refers to the result of a *transformation* of a figure.

## independent and dependent variables

In mathematical modelling, the independent variable is a measurable or observable quantity that has a relation to (or a causal effect on) one or more other quantities, called the dependent variables.

For example, a scientific investigation considers the relationship between the amount of water supplied and the growth of a plant. It is assumed that there is a causal link between the two quantities. A choice is made to make the amount of water the independent variable, because it is the quantity whose effect is to be investigated, thus making the growth of the plant the dependent variable.

When graphing the results of such an investigation, the convention is to display the independent variable (the amount of water) on the horizontal axis and the dependent variable (the growth of the plant) on the vertical axis.

## independent event

Two events are *independent* if knowing the outcome of one event tells us nothing about the outcome of the other event.

## index laws

Index laws are rules for manipulating indices. They include

$$(x^a x^b = x^{a+b}) \quad (\text{left}(x^a) \text{right})^b = x^{ab}$$

$$(x^a y^a = (xy)^a)$$

and

$$(x^0 = 1) \quad (x^{-a} = \frac{1}{x^a}) \quad (x^{1/a} = \sqrt[a]{x})$$

## index notation

When the product of  $(a \times a \times a)$  is written as  $(a^3)$ , the number 3 is called the index, often also referred to as the ‘power’ or the ‘exponent’.

## indices

(plural) See *index*.

## inequality

An *inequality* is a statement that one number or algebraic expression is less than (or greater than) another.

There are five types of inequalities:

The relation a is less than b is written  $a < b$

a is greater than b is written  $a > b$

a is less than or equal to b is written  $a \leq b$

a is greater than or equal to b is written  $a \geq b$ .

a is unequal to b is written  $a \neq b$ .

## informal unit

*Informal units* are not part of a standardised system of units for measurement; for example, an informal unit for length could be paperclips of uniform length. An informal unit for *area* could be uniform paper squares of any size. Informal units are sometimes referred to as non-standard units.

## integer

The *integers* are the “*whole numbers*” including those with negative sign  $\dots -3, -2, -1, 0, 1, 2, 3 \dots$ . In Latin, the word *integer* means “whole.” The set of integers is usually denoted by Z. Integers are basic building blocks in mathematics.

## interquartile range

The *interquartile range* (IQR) is a measure of the spread within a *numerical data set*. It is equal to the upper quartile ( $Q_3$ ) minus the lower quartile ( $Q_1$ ); that is,  $IQR = Q_3 - Q_1$ .

The IQR is the width of an interval that contains the middle 50% (approximately) of the data values. To be exactly 50%, the *sample size* must be a multiple of four.

## interval

An *interval* is a *subset* of the *number line*.

## irrational number

An irrational number is a real number that is not rational, that means, it cannot be represented as a fraction. Some commonly used irrational numbers are  $\pi$  and  $\sqrt{2}$ .

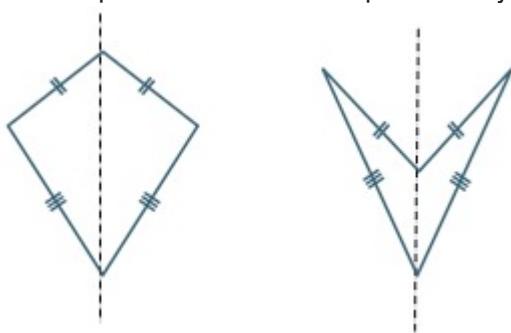
Decimal representations of irrational numbers are non-terminating. For example, the Euler Number e is an irrational real number whose decimal expansion begins  $e=2.718281828\dots$ .

## irregular shape

An *irregular shape* is a shape where not all sides and angles are equal in length or magnitude. By contrast, a *regular shape* has sides and *angles* that are equal in length and magnitude; for example, a square is a regular shape, while a scalene triangle is irregular.

## kite

A kite is a quadrilateral with two pairs of adjacent sides equal.



A kite may be convex as shown in the diagram above to the left or non-convex as shown above to the right. The axis of symmetry of the kite is shown.

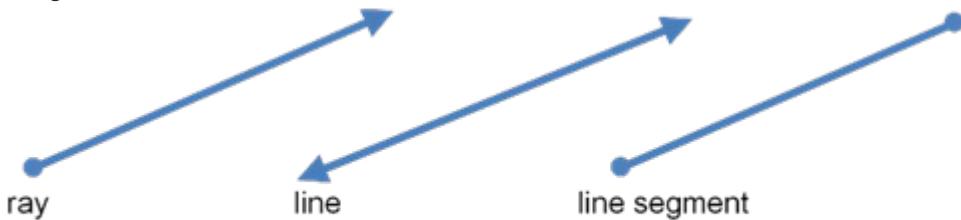
## leading term

The leading term of a polynomial is the term that contains the variable raised to the highest power. For example, in the polynomial  $(3x^2-5x+2)$ , the leading term is  $(3x^2)$ .

## line

In geometry, a line extends infinitely in both directions.

A line is different from a ray (which extends from a point toward infinity) and a line segment (which extends between two points). Lines are depicted with arrow heads on both ends to distinguish them from rays and line segments.



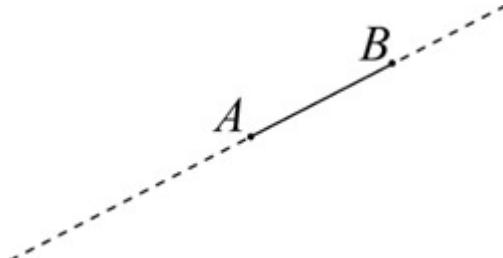
## linear equation

A linear equation is an equation involving just linear terms, that is, no variables are raised to a power greater than one. The general form of a linear equation in one variable is  $(ax+b=0)$ , where  $a$  and  $b$  cannot both be 0. The solution of a linear equation in general form is  $(x=\frac{-b}{a})$ . A linear equation with two variables takes the general form  $(ax+by+c=0)$ , where  $a$  and  $b$  cannot both be 0. If  $a \neq 0$ , then the point where the line intersects the  $x$ -axis (the  $x$ -intercept) is  $(\frac{c}{a})$ . If  $b \neq 0$ , then the point where the line intersects the  $y$ -axis (the  $y$ -intercept) is  $(\frac{c}{b})$ . Two-variable, or two-dimensional linear equations also come in the form  $(y=mx+b)$ . The constant  $m$  indicates the gradient or slope of a line, while  $b$  represents the  $y$ -intercept.

## line segment

If  $A$  and  $B$  are two points on a line, the part of the line between and including  $A$  and  $B$  is called a line segment or interval.

The distance  $AB$  is a measure of the length of  $AB$ .



## location

In Measurement and Geometry, the relative position of an object is referred to as its *location*. It can be expressed in terms of a description (such as ‘next to’, ‘behind’, ‘on top of’), a grid reference (such as ‘A5’), or a coordinate (such as ‘(-2,7)').

In Statistics and Probability, a measure of *location* is a single number that can be used to indicate a central or ‘typical value’ within a set of data. The most commonly used measures of location are the *mean* and the *median* although the *mode* is also sometimes used for this purpose.

## logarithm

The logarithm of a positive number  $x$  is the power to which a given number  $b$ , called the base, must be raised in order to produce the number  $x$ . The logarithm of  $x$ , to the base  $b$  is denoted by  $\log_b x$ . Algebraically, the statements  $\log_b x = y$  and  $b^y = x$  are equivalent in the sense that both statements express the identical relationship between  $x$ ,  $y$  and  $b$ . For example,  $\log_{10} 100 = 2$  because  $10^2 = 100$ , and  $\log_2 \left(\frac{1}{32}\right) = -5$  because  $2^{-5} = \frac{1}{32}$ .

## mass

*Mass* is the measure of how much matter is in a person, object, or substance. Mass is measured in grams, kilograms, tonnes, ounces, or pounds. It is distinct from weight, which refers to the amount of gravitational force acting on matter. If you travelled to Mars, your mass would be the same as it was on Earth, but your weight would be less due to the weaker gravitational force on Mars.

## measures of central tendency

In statistics, the term *measures of central tendency* refers to different methods of calculating typical values (commonly called averages) within a set. The most commonly used measures of central tendency are the *mean*, *median*, and *mode*.

### mean

The arithmetic mean of a list of numbers is the sum of the data values divided by the number of numbers in the list.

In everyday language, the arithmetic mean is commonly called the average; for example, for the following list of five numbers, {2, 3, 3, 6, 8}, the mean equals  $(2+3+3+6+8)/5 = 22/5 = 4.4$ .

### median

The *median* is the value in a set of ordered data that divides the *data* into two parts. It is frequently called the ‘middle value’.

Where the number of observations is odd, the median is the middle value; for example, for the following ordered data set with an odd number of observations, the median value is five.

1 3 3 4 5 6 8 9 9

Where the number of observations is even, the median is calculated as the *mean* of the two central values; for example, in the following ordered data set, the two central values are 5 and 6, and median value is the mean of these two values, 5.5.

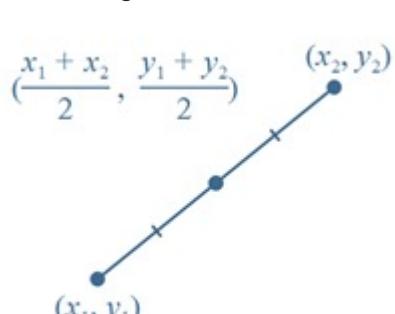
1 3 3 4 5 6 8 9 9 10

The median provides a measure of location of a data set that is suitable for both *symmetric* and *skewed* distributions and is also relatively insensitive to *outliers*.

## midpoint

The midpoint M of a line segment AB, which extends between points A and B, is the point that divides the segment into two equal parts.

If  $(A(x_1, y_1))$  and  $(B(x_2, y_2))$  are points on a Cartesian plane, then the midpoint M of the line segment AB has coordinates  $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$



## mode

The *mode* is a measure calculated by identifying the value that appears with greatest *frequency* in a *set of data*. If two numbers occur in a set with equal frequency, the set is said to contain *bimodal data*. If there are more than two numbers in a set that occur with equal frequency, the set is said to contain *multimodal data*. The mode of the set {1, 2, 3, 4, 4, 5} is 4. In the set {1, 2, 2, 4, 5, 7, 7}, the modes are both 2 and 7, making the set bimodal. The mode is sometimes used as a measure of *location*.

## monic

A monic polynomial is one in which the coefficient of the leading term is 1. For example,  $(x^3+2x^2-7)$  is monic, but  $(4x^2-x+1)$  is not.

## multimodal data

*Multimodal data* is *data* whose distribution has more than two *modes*.

## multiples

A multiple of a whole number is the product of that number and an integer.

A multiple of a real number  $(x)$  is any number that is a product of  $(x)$  and an integer; for example, 4.5 and -13.5 are multiples of 1.5 because  $4.5=3\times1.5$  and  $-13.5=-9\times1.5$ .

## natural number

A *natural number* can refer either to a *positive integer* (which excludes negative numbers and zero) or a *counting number* (which excludes negative numbers but includes zero). The *set of natural numbers* is usually denoted by N.

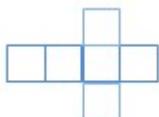
## negative integer

A *negative integer* is any *integer* whose value is below zero. That is, -1, -2, -3, -4, -5, -6 ⋯.

## net

A net is a plane figure that can be folded to form a polyhedron.

One possible net for a cube is shown



## non-monic

A polynomial in one variable is said to be non-monic, if the coefficient of the leading term is unequal to one. For example,  $(2x^3+2x^2+3x-4)$  is a non-monic polynomial, whereas  $(x^3+2x^2+3x-4)$  is a monic polynomial.

## non-negative integers

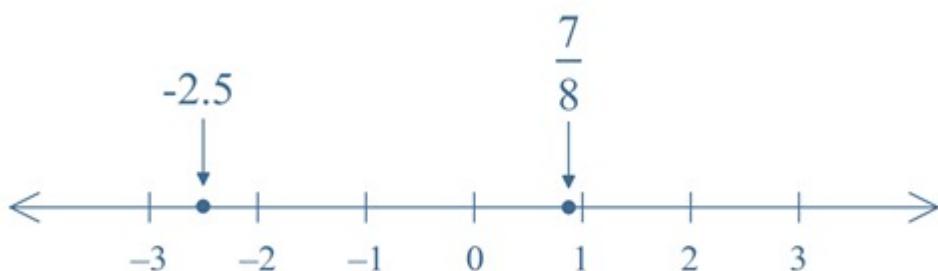
A *non-negative integer* is an *integer* that is not negative, and is either zero or positive. It differs from a *positive integer*, which excludes zero. Non-negative integers are 0, 1, 2, 3, 4, 5⋯.

## non-zero whole numbers

*Non-zero whole numbers* are *whole numbers* that explicitly exclude zero. Non-zero whole numbers are 1, 2, 3, 4, 5, 6⋯.

## number line

A number line, like the one below, gives a pictorial representation of real numbers. An example is given below depicting the location of a negative decimal and a positive fraction.



## number sentence

A *number sentence* is typically an equation or inequality expressed using numbers and common symbols; for example,  $10 + 10 = 3 + 7 + 5 + 5$  could describe a situation where 2 packets of 10 coloured pens contained 3 red, 7 green, 5 yellow and 5 white.

## numerator

In the *fraction*  $\frac{a}{b}$ ,  $a$  is the numerator. If an object is divided into  $b$  equal parts, then the fraction  $\frac{a}{b}$  represents  $a$  of these parts taken together; for example, if a line segment is divided into 5 equal parts, each of those parts is one fifth of the whole and 3 of these parts taken together corresponds to the fraction  $\frac{3}{5}$ .

## numeral

A *numeral* is a figure or symbol used to represent a number; for example,  $-3$ ,  $0$ ,  $45$ ,  $\text{IX}$ ,  $\pi$ .

## numerical data

*Numerical data* is data associated with a *numerical variable*.

Numerical variables are variables whose values are numbers, and for which arithmetic processes such as adding and subtracting, or calculating an average, make sense.

## obtuse angle

An *obtuse angle* is bigger than  $90^\circ$  but smaller than  $180^\circ$ .

## odd number

An *odd number* is an *integer* that is not *divisible* by 2. The odd numbers are  $\dots, -5, -3, -1, 1, 3, 5, \dots$

## operation

*Operation* is the process of combining numbers or expressions. In the primary years, operations include addition, subtraction, multiplication, and division. In later years, operations include, for instance, raising to a power, taking the logarithm, and more complex operations, such as integration.

## ordered pair

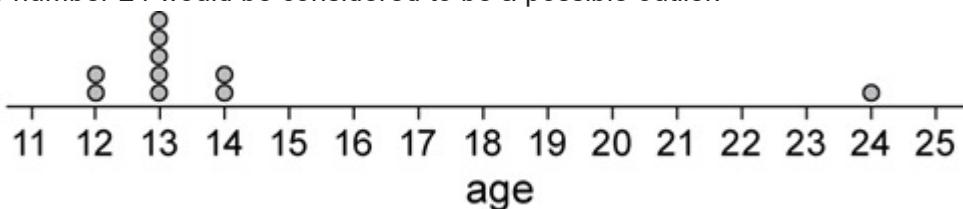
In mathematics, an *ordered pair* is a collection of two numbers whose order is significant. Ordered pairs are used to describe the location of a point in the *Cartesian plane*.

## order of operations

*Order of operations* refers to a collection of rules for simplifying expressions. It stipulates that calculations in brackets must be made first, followed by calculations involving *indices (powers, exponents)*, then multiplication and division (working from left to right), and lastly, addition and subtraction (also in order from left to right); for example, in  $5-6\div 2+7$ , the division is performed first and the expression becomes  $5-3+7=9$ . If the convention is ignored and the *operations* are performed in the order they are written, the incorrect result, 6.5 is obtained.

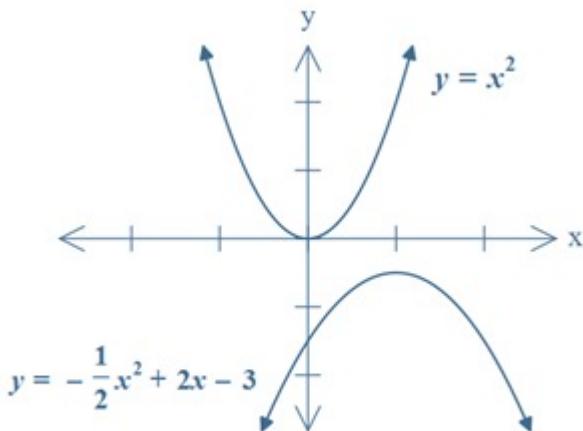
## outlier

An outlier is a data value that appears to stand out from the other members of the data set by being unusually high or low. The most effective way of identifying outliers in a data set is to graph the data; for example, in the following list of ages of a group of 10 people, {12, 12, 13, 13, 13, 13, 13, 13, 14, 14, 24}, the number 24 would be considered to be a possible outlier.



## parabola

In algebra, a parabola is the graph of a function of the general form  $y=ax^2+bx+c$ , where  $a$ ,  $b$  and  $c$  are real numbers and  $a \neq 0$ . Two examples of parabolas are shown below.

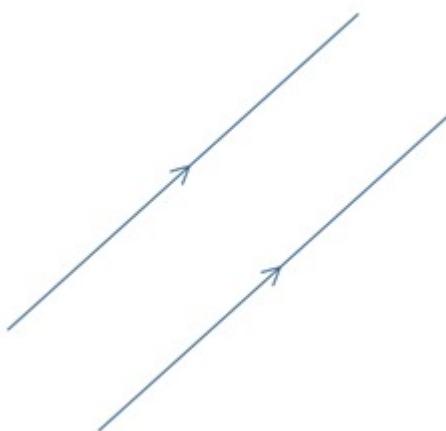


## parallel

Parallel lines are lines in a plane which do not intersect or touch each other at any point. Parallel lines can never intersect, even if they were to continuously extend toward infinity.

Two lines are parallel, if they have the same gradient (or slope).

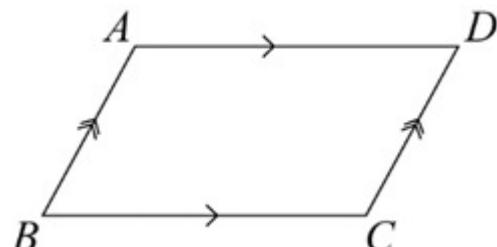
The lines below are parallel to one another, as indicated by the use of the arrow signs. In text, the symbol  $\parallel$  is used to denote parallel lines; for example,  $a \parallel b$  is read as “line a is parallel to line b”.



## parallelogram

A parallelogram is a quadrilateral whose opposite sides are parallel.

Thus the quadrilateral ABCD shown below is a parallelogram because  $AB \parallel DC$  and  $AD \parallel BC$ .



Properties of a parallelogram

- The opposite angles of a parallelogram are equal.
- The opposite sides of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.

## partitioning

*Partitioning* means dividing a quantity into parts. In the early years, it commonly refers to the ability to think about numbers as made up of two parts, such as, 10 is 8 and 2. In later years it refers to dividing both continuous and discrete quantities into equal parts.

## percentage

A percentage is a fraction whose denominator is 100; for example, 6 percent (written as 6%) is the percentage whose value is  $\frac{6}{100}$ .

Similarly, 40 as a percentage of 250 is  $\frac{40}{250} \times 100 = 16\%$ .

## percentile

*Percentiles* are the 99 values that divide an ordered *data set* into 100 (approximately) equal parts. It is only possible to divide a data set into exactly 100 equal parts when the number of data values is a multiple of one hundred.

Within the above limitations, the first percentile divides off the lower 1% of data values. The second, the lower 2% and so on. In particular, the lower *quartile* ( $Q_1$ ) is the 25<sup>th</sup> percentile, the *median* is the 50<sup>th</sup> percentile and the upper quartile is the 75<sup>th</sup> percentile.

Percentiles are often used to report comparative test results. A student who scores in the 90<sup>th</sup> percentile for a given test has scored higher than 90% of other students who took the test. A student who scores in the 10<sup>th</sup> percentile would have scored better than only 10% of students who took the test.

## perimeter

The *perimeter* of a plane figure is the length of its boundary. The perimeter of a figure can be calculated by adding the lengths of all its sides.

## perpendicular

In geometry, two lines are said to be *perpendicular* to each other, if they meet at a *right angle* (90 degrees).

## Pi

Pi is the name of the Greek letter  $\pi$  that is used to denote the ratio of the circumference of any circle to its diameter. The number  $\pi$  is irrational, but  $\frac{22}{7}$  is a rational approximation. The decimal expansion of  $\pi$  begins:

$\pi = 3.14159265358979\dots$

## picture graphs

A picture graph is a statistical graph for organising and displaying categorical data.

### Ball sports played by students in Year 4

Football	   
Basketball	  
Netball	    
Soccer	 
Rugby	  
Hockey	 
Key	 = 10 Students

## place value

Place value refers to the value of a digit as determined by its position in a number, relative to the ones (or units) place. For integers, the ones place is occupied by the rightmost digit in the number; for example, in the number 2 594.6 the 4 denotes 4 ones, the 9 denotes 90 ones or 9 tens, the 5 denotes 500 ones or 5 hundreds, the 2 denotes 2000 ones or 2 thousands, and the 6 denotes  $\frac{1}{10}$  of a one or 6 tenths.

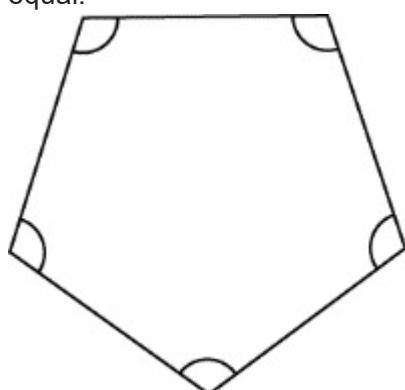
## point

A *point* marks a position, but has no size.

## polygon

A polygon is a plane figure bounded by three or more line segments. The word derives from Greek polys “many” and gonia “angle”.

The regular pentagon shown below is an example of a polygon. It is called a pentagon because it has five sides (and five angles). It is called regular because all sides have equal length and all interior angles are equal.



## polyhedron

A polyhedron is a three-dimensional object, or a solid, which consists of a collection of polygons, joined at their edges and making up the faces of the solid. The word derives from Greek polys “many” and hedra “base” or “seat”.

The solid below is an example of a polyhedron (called an icosahedra and consisting of 20 faces).



## polynomial

A polynomial in one variable  $\$x\$$  is a finite sum of terms of the form  $\$ax^k\$$ , where  $\$a\$$  is a real number and  $\$k\$$  is a non-negative integer.

A non-zero polynomial can be written in the form  $\$a_0+a_1x+a_2x^2+\dots+a_nx^n\$$ , where  $\$n\$$  is a non-negative integer and  $\$a_n \neq 0\$$ .

The term that contains the variable  $\$x\$$  raised to the highest power, that is  $\$a_nx^n\$$ , is called the leading term.

The numbers  $\$a_0, a_1, \dots, a_n\$$  are called the coefficients of the terms. Coefficients include the preceding sign.

For example, in the polynomial  $\$3x^2-5x+2\$$ , the leading term is  $\$3x^2\$$  and the coefficient of the second term is  $\$-5\$$ .

## population

A *population* is the complete *set* of individuals, objects, places etc. about which we want information. A *census* is an attempt to collect information about the whole population.

## positive integer

A *positive integer* is an *integer* that excludes negative numbers and zero. Positive integers are 1, 2, 3, 4, 5, 6, ....

## primary data

*Primary data* is original *data* collected by the user. Primary data might include data obtained from interviews the user has conducted herself, or observations the user has made during an experiment.

## prime factor

A *prime factor* of a number is a *factor* of that number which is prime.

## prime number

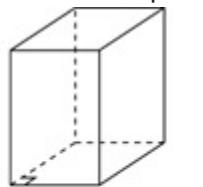
A *prime number* is a *natural number* greater than 1 that has no *factor* other than 1 and itself.

## prism

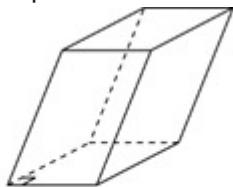
A prism is a polyhedron that has two congruent and parallel faces and all its remaining faces are parallelograms.

A right prism is a polyhedron that has two congruent and parallel faces and all its remaining faces are rectangles. A prism that is not a right prism is often called an oblique prism.

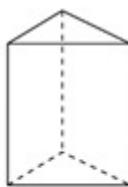
Some examples of prisms are shown below.



Right  
rectangular  
prism



Oblique  
rectangular  
prism



Right  
triangular  
prism

## probability

The *probability* of an *event* is a number between 0 and 1 that indicates the chance of that event happening; for example, the probability that the sun will come up tomorrow is 1, the probability that a fair coin will come up ‘heads’ when tossed is 0.5, while the probability of someone being physically present in Adelaide and Brisbane at exactly the same time is zero.

## product

A product is the result of multiplying together two or more numbers or algebraic expressions; for example, 36 is the product of 9 and 4, and  $\$x^2-y^2\$$  is product of  $\$x-y\$$  and  $\$x+y\$$ .

## proof

A *proof* is a rigorous mathematical argument that demonstrates the truth of a given proposition. A mathematical statement that has been established by means of a proof is called a *theorem*.

## proportion

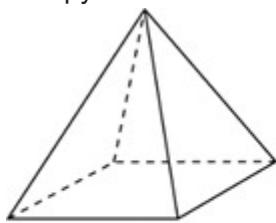
Two quantities are in *proportion*, if there is a constant *ratio* between them.

## protractor

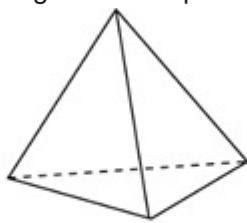
A *protractor* is an instrument for measuring *angles*. It uses degrees as the unit of measurement and is commonly in the shape of a semi-circle ( $180^\circ$ ) or circle ( $360^\circ$ ).

## pyramid

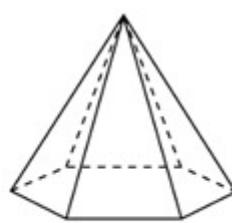
A pyramid is a polyhedron with a polygonal base and triangular sides that meet at a point called the vertex. The pyramid is named according to the shape of its base.



square-based pyramid



triangular-based pyramid



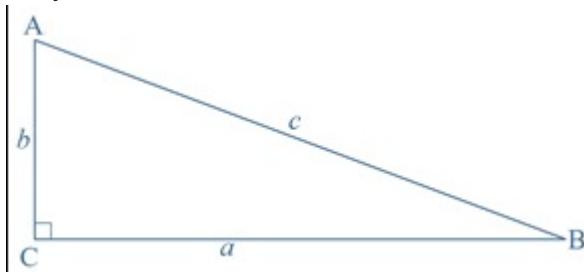
hexagonal-based pyramid

## Pythagoras' theorem

Pythagoras' theorem states that for a right-angled triangle:

The square of the hypotenuse of a right-angled triangle equals the sum of the squares of the lengths of the other two sides.

In symbols,  $c^2 = a^2 + b^2$ .

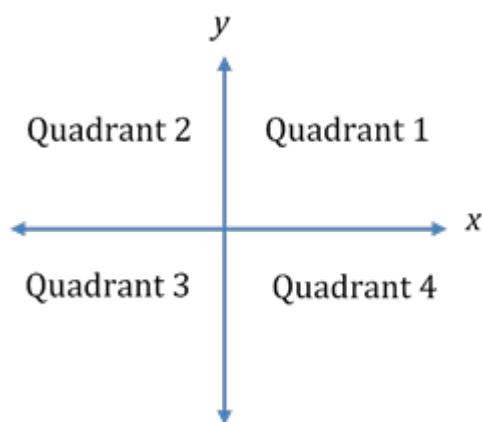


The converse

If  $c^2 = a^2 + b^2$  in a triangle ABC, then  $\angle C$  is a right angle.

## quadrant

Quadrant refers to the four sections of the Cartesian plane created through the intersection of the x and y-axes. They are numbered 1 through 4, beginning with the top right quadrant and moving counter clockwise around the plane. Each of the four quadrants is labelled on the plane below.



## quadratic equation

The general quadratic equation in one variable is  $ax^2+bx+c=0$ , where  $a \neq 0$ .

The solutions are given by the quadratic formula:  $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ .

## quadratic expression

A quadratic expression or function contains one or more terms in which the variable is raised to the second power, but no variable is raised to a higher power. Examples of quadratic expressions include  $3x^2+7$  and  $x^2+2xy+y^2-2x+y+5$ .

## quadrilateral

A *quadrilateral* is a polygon with four sides.

## quartile

*Quartiles* are the values that divide an ordered *data set* into four (approximately) equal parts. It is only possible to divide a data set into exactly four equal parts, when the number of data values is a *multiple* of four.

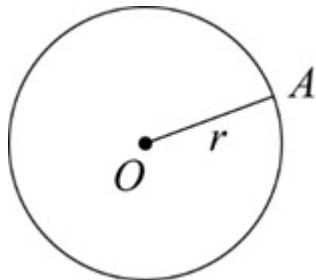
There are three quartiles. The first, the *lower quartile* ( $Q_1$ ) divides off (approximately) the lower 25% of data values. The second quartile ( $Q_2$ ) is the *median*. The third quartile, the *upper quartile* ( $Q_3$ ), divides off (approximately) the upper 25% of data values.

## quotient

A *quotient* is the result of dividing one number or *algebraic expression* by another. See also *remainder*.

## radius

The radius of a circle ( $r$ ) is the distance from its centre to any point ( $A$ ) on its perimeter, and is equal to half of the circle's diameter.



Putting the point of a pair of compasses at the centre and opening the arms to the radius can draw a circle.

## random number

A *random number* is one whose value is governed by chance; such as, the number of dots showing when a fair die is tossed. The value of a random number cannot be predicted in advance.

## random sample

A sample is called a *random sample* (or a *simple random sample*), if it is selected from a *population* at random. That is, all the *elements* of the population had an equal *probability* of being included in the sample.

## range

In statistics, the *range* is the difference between the largest and smallest observations in a *data set*.

The range can be used as a measure of spread in a data set, but it is extremely sensitive to the presence of *outliers* and should only be used with care.

## ratio

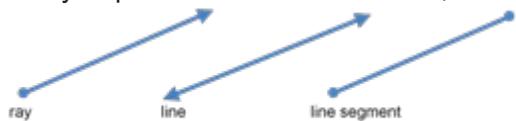
A ratio is a quotient of two numbers, magnitudes, or algebraic expressions. It is often used as a measure of the relative size of two objects; for example, the ratio of the length of a side of a square to the length of a diagonal is  $\sqrt{2}$ ; that is,  $\frac{1}{\sqrt{2}}$ .

## rational numbers

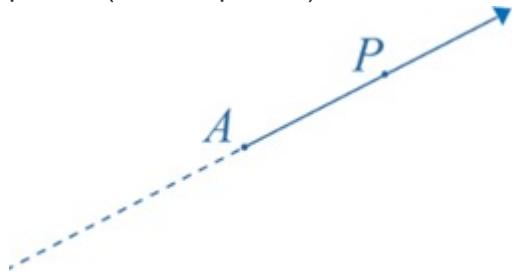
The *rational numbers* are the set of all numbers that can be expressed as fractions, that is, as quotients of two *integer* values. The *decimal expansion* of a rational number is either *terminating* or *recurring*.

## ray

A ray is the part of a line that starts at a point and continues in a particular direction to infinity. Rays are usually depicted with an arrow head, which indicates the direction in which the line continues to infinity.



Any point A on a line divides the line into two pieces called rays. The ray AP is that ray which contains the point P (and the point A) and extends toward infinity. The point A is called the vertex of the ray.



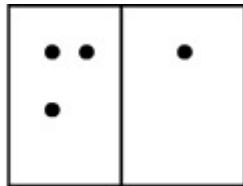
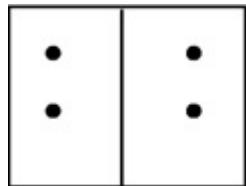
## real number

The numbers generally used in mathematics, in scientific work and in everyday life are the *real numbers*. They can be pictured as *points* on a *number line*, with the integers evenly spaced along the line, and a real number  $b$  to the right of a real number  $a$  if  $b > a$ .

A real number is either *rational* or *irrational*. Every real number has a *decimal* expansion. Rational numbers are the ones whose decimal expansions are either *terminating* or *recurring*, while irrational numbers can only be approximated in the decimal number system.

## rearranging parts

Rearranging parts refers to moving counters, numbers, etc., in order to change the visual representation of the number; for example, '4' could be represented as either of the two combinations below.

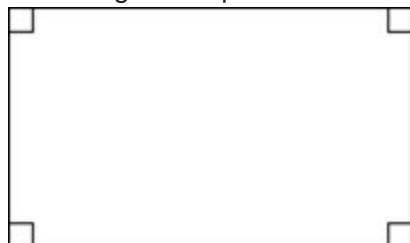


## reasonableness

*Reasonableness* refers to how appropriate an answer is. “Does this answer make sense?” and “Does this answer sound right?” are two questions that should be asked when thinking about reasonableness.

## rectangle

A rectangle is a quadrilateral in which all angles are right angles.

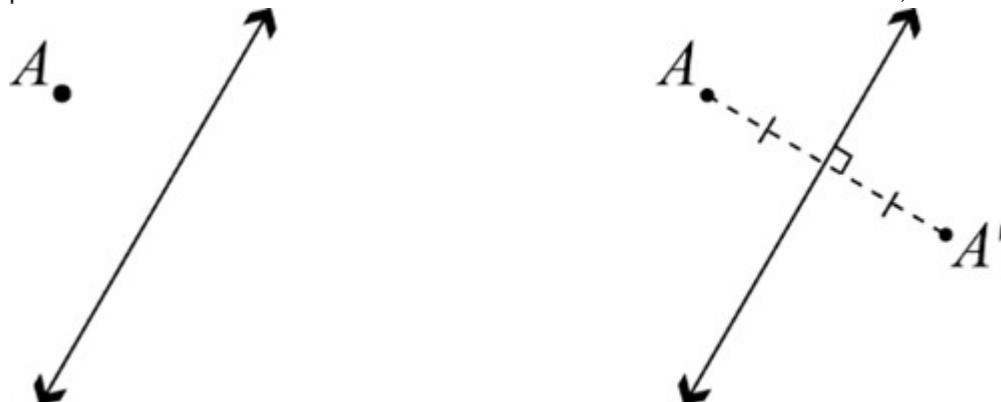


## reflex angle

A *reflex angle* is an *angle* with a size that is larger than  $180^\circ$  but smaller than  $360^\circ$ .

## reflection

To reflect the point A in an axis of reflection, a line is drawn at right angles to the axis of reflection and the point A' is marked at the same distance from the axis of reflection as A, but on the other side.



The point A' is called the reflection image of A.

A reflection is a transformation that moves each point to its reflection image.

## regular shape

A *regular shape* has sides and *angles* that are equal in length and magnitude.

## related denominators

Related denominators occur where one denominator is a multiple of the other; for example, the fractions  $\frac{1}{3}$  and  $\frac{5}{9}$  have related denominators because 9 is a multiple of 3.

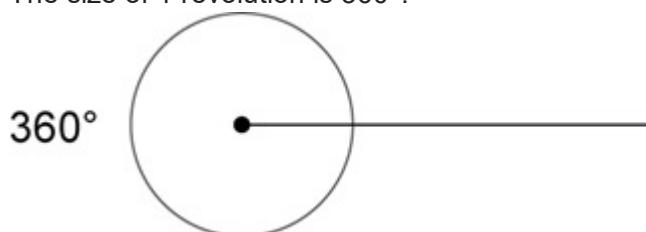
Fractions with related denominators are more easily added and subtracted than fractions with unrelated denominators, because only one needs to be rewritten; for example, to add  $\frac{1}{3}$  and  $\frac{5}{9}$  we can rewrite  $\frac{1}{3}$  as the equivalent fraction  $\frac{3}{9}$  and then compute  $\frac{3}{9} + \frac{5}{9} = \frac{8}{9}$ .

## remainder

A *remainder* is the amount left over when one number or algebraic quantity a is divided by another b. If a is divisible by b then the remainder is 0. For example, when 68 is divided by 11, the remainder is 2, because 68 can be expressed as  $68=6\times11+2$ .

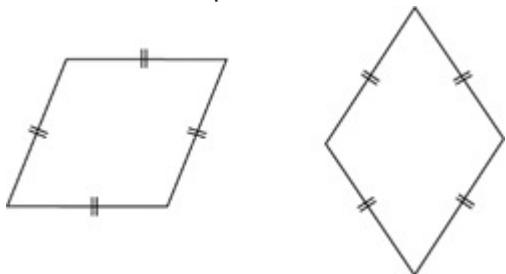
## revolution

A revolution is the amount of turning required to rotate a ray about its endpoint until it falls back onto itself. The size of 1 revolution is  $360^\circ$ .



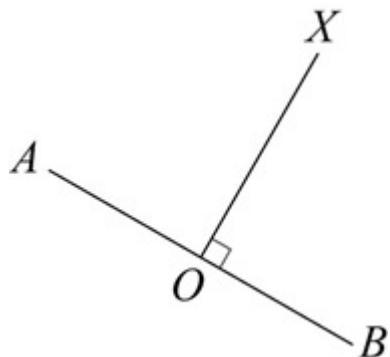
## rhombus

A rhombus is a quadrilateral with all sides equal.



## right angle

A right angle is half a straight angle, and so is equal to  $90^\circ$ .



## rounding

The decimal expansion of a real number is rounded when it is approximated by a terminating decimal that has a given number of decimal digits to the right of the decimal point.

Rounding to  $n$  decimal places is achieved by removing all decimal digits beyond (to the right of) the  $\$\$n^{\{th\}}\$$  digit to the right of the decimal place, and adjusting the remaining digits where necessary.

If the first digit removed (the  $\$\{(n+1)\}^{\{th\}}\$$  digit) is less than 5 the preceding digit is not changed; for example, 4.02749 becomes 4.027 when rounded to 3 decimal places.

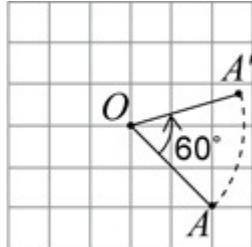
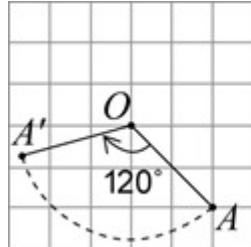
If the first digit removed is greater than or equal to 5, then the preceding digit is increased by 1; for example, 6.1234586 becomes 6.12346 when rounded to 5 decimal places.

## rotation

In a plane, a rotation is a transformation that turns a figure about a fixed point, called the centre of rotation. A rotation is specified by:

- the centre of rotation O
- the angle of rotation
- the direction of rotation (clockwise or counter-clockwise).

In the first diagram below, the point A is rotated through  $120^\circ$  clockwise about O. In the second diagram, it is rotated through  $60^\circ$  counter-clockwise about O.



A rotation is a transformation that moves each point to its rotation image.

## sample

A *sample* is part of a *population*. It is a *subset* of the population, often randomly selected for the purpose of estimating the value of a characteristic of the population as a whole.

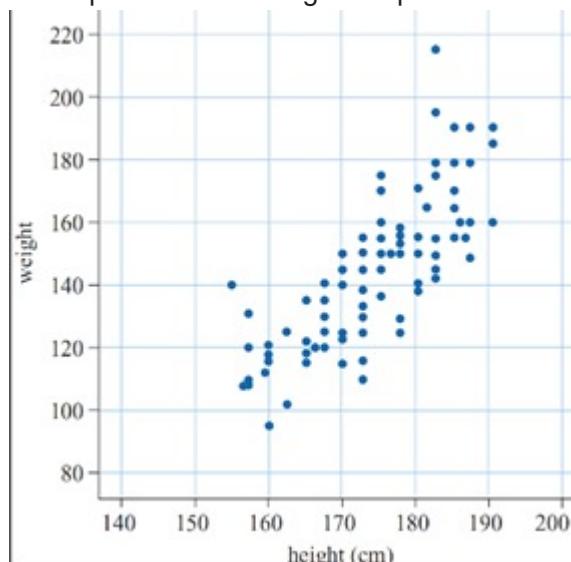
For instance, a randomly selected group of eight-year old children (the sample) might be selected to estimate the incidence of tooth decay in eight-year old children in Australia (the population).

## sample space

A *sample space* is the *set* of all possible outcomes of a *chance experiment*; for example, the set of outcomes (also called *sample points*) from tossing two heads is {HH, HT, TH, TT}, where H represents a ‘head’ and T a ‘tail’.

## scatter plots

When two variables are numerical then a scatter plot (or bivariate plot) may be constructed. This is an important tool in the analysis of bivariate data, and should always be examined before further analysis is undertaken. The pairs of data points are plotted on a Cartesian plane, with each pair contributing one point to the plot. The following example examines the features of the scatterplot in more detail.



Suppose we record the heights and weights of a group of 100 people. The scatterplot of those data would be 100 points. Each point represents one person's height and weight.

## scientific notation

Scientific notation is a distinct way of writing numbers that are too big or too small to be written in an accessible way. Numbers are expressed as a product of the power of 10 and a decimal that has just one digit to the left of the decimal point; for example, the scientific notation for 34,590 is  $3.459 \times 10^4$ , and the scientific notation for 0.000004567 is  $4.567 \times 10^{-6}$ .

Many electronic calculators will show these as 3.459E4 and  $4.567 \times 10^{-6}$ .

## secondary data

*Secondary data* is data collected by others. Sources of secondary data include, web-based data, the media, books, scientific papers, etc.

## sequence

A *sequence* is an ordered collection of *elements*. When written, the elements are separated by commas. Sequences can be finite (e.g., 1, 2, 3, 4), or infinite (1, 2, 3, 4, 5, 6...).

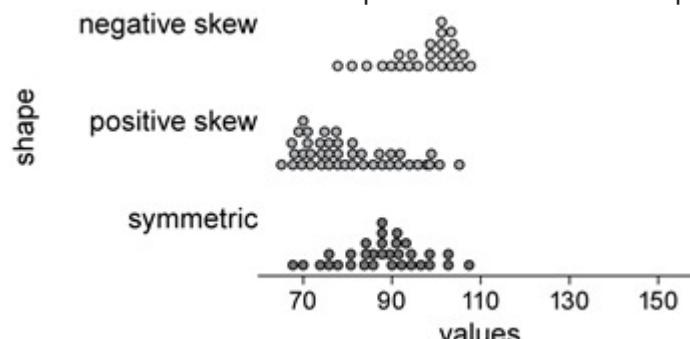
## set

In probability and statistics, a *set* is a well-defined collection of objects, *events* or outcomes. Each item within a set is called an *element* of the set.

## shape

The shape of a numerical data distribution is mostly simply described as symmetric, if it is roughly evenly spread around some central point or skewed, if it is not. If a distribution is skewed, it can be further described as positively skewed ('tailing-off' to the upper end of the distribution) or negatively skewed ('tailing-off' to the lower end of the distribution).

These three distribution shapes are illustrated in the parallel dot plot display below.

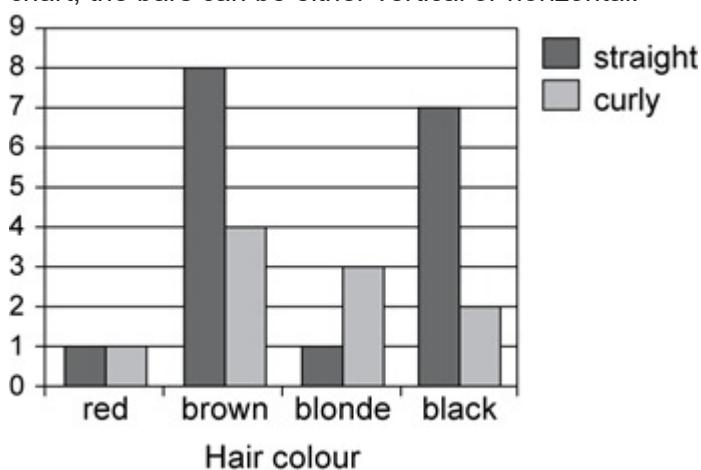


Dot plots, histograms and stem plots can all be used to investigate the shape of a data distribution.

## side-by-side column graph

A side-by-side column graph can be used to organise and display the data that arises when a group of individuals or things are categorised according to two or more criteria; for example, the side-by-side column graph below displays the data obtained when 27 children are categorised according to hair type (straight or curly) and hair colour (red, brown, blonde, black). The legend indicates that blue columns represent children with straight hair and red columns children with curly hair.

Side-by-side column graphs are frequently called side-by-side bar graphs or bar charts. In a bar graph or chart, the bars can be either vertical or horizontal.



## similarity

**Similarity (general):**

Two plane figures are called *similar* if an *enlargement* of one figure is *congruent* to the other. That is, if one can be mapped to the other by a sequence of *translations*, *rotations*, *reflections* and *enlargements*.

Similar figures thus have the same shape, but not necessarily the same size.

**Similarity (triangles):**

There are four standard tests to determine if two triangles are similar

**AAA:** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

**SAS:** If the ratio of the lengths of two sides of one triangle is equal to the ratio of the lengths of two sides of another triangle, and the included angles are equal, then the two triangles are similar.

**SSS:** If we can match up the sides of one triangle with the sides of another so that the ratios of matching sides are equal, then the two triangles are similar.

**RHS:** If the ratio of the hypotenuse and one side of a right-angled triangle is equal to the ratio of the hypotenuse and one side of another right-angled triangle, then the two triangles are similar.

## simple interest

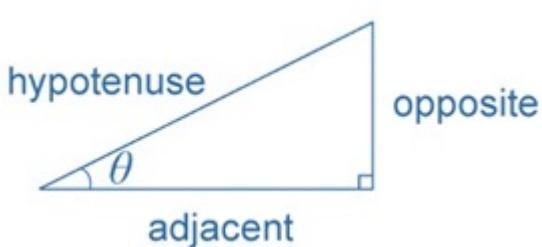
Simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal; for example, if the principle  $\$P$  earns simple interest at the rate of  $i\%$  per period, then after  $n$  periods the accumulated simple interest is  $\$Pni/100$ .

## simultaneous equations

Two or more *equations* form a set of *simultaneous equations* if there are conditions imposed simultaneously on all of the *variables* involved.

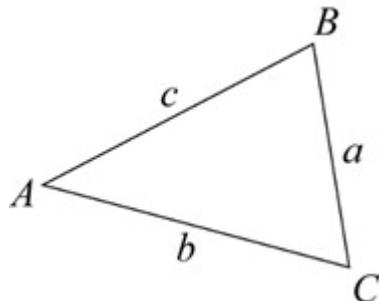
## sine

In any right-angled triangle, the sine of an angle is defined as the length of the side opposite the angle divided by the length of the hypotenuse;  $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ , where  $0^\circ < \theta < 90^\circ$ .



## sine rule

In any triangle ABC,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ .



In words it says:

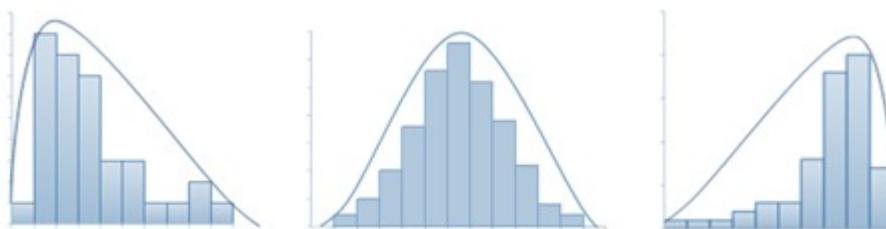
Any side of a triangle over the sine of the opposite angle equals any other side of the triangle over the sine of its opposite angle.

## skewness

Skewness is a measure of asymmetry (non-symmetry) in a distribution of values about the mean of a set of data.

In the diagrams below, the histogram to the left is positively skewed. Data values are concentrated at the beginning of the number line, causing the graph to have a long tail to the right and a very short tail to the left. The histogram to the right is negatively skewed. Data values are concentrated further right along the number line, causing the graph to have a long tail to the left and a very short tail to the right. The mode, median and mean will not coincide.

When the distribution of values in a set of data is symmetrical about the mean, the data is said to have normal distribution. The histogram in the middle is normally distributed.



## skip counting

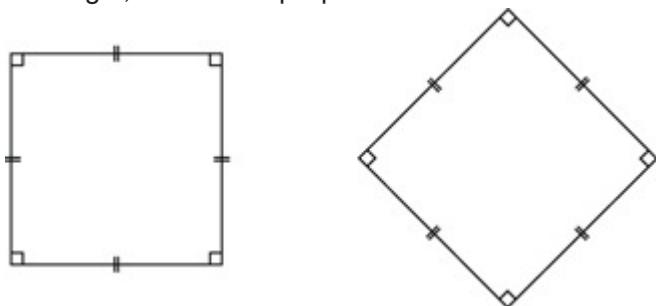
*Skip counting* is counting by a number that is not 1; for example, skip counting forwards by 2 would be 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 ... Skip counting backwards by 3 from 21 would be 21, 18, 15, 12, 9, 6, 3, 0.

## solid

A *solid* is any *three-dimensional* geometrical figure.

## square

A square is a quadrilateral that is both a rectangle and a rhombus. A square thus has all the properties of a rectangle, and all the properties of a rhombus.



## standard deviation

*Standard deviation* is a measure of the variability or spread of a *data set*. It gives an indication of the degree to which the individual data values are spread around their *mean*.

## stem and leaf plot

A stem-and-leaf plot is a method of organising and displaying numerical data in which each data value is split into two parts, a ‘stem’ and a ‘leaf’; for example, the stem-and-leaf plot below displays the resting pulse rates of 19 students.

### pulse rate

6		8	8	8	9
7		0	1	1	4
		6	6	6	8
8		2	6	8	8
9		0	6		
10		4			
11		0			

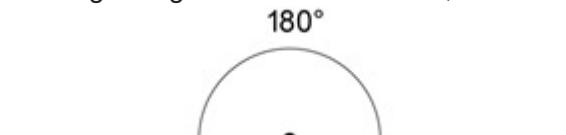
In this plot, the stem unit is ‘10’ and the leaf unit is ‘1’. Thus the top row in the plot 6 | 8 8 8 9 displays pulse rates of 68, 68, 68 and 69.

## stemplot

*Stemplot* is a synonym for *stem-and-leaf plot*.

## straight angle

A straight angle is half a revolution, and so is equal to  $180^\circ$ .



## subitising

*Subitising* refers to the recognition of the number of objects in a collection without consciously counting.

## subset

In probability and statistics, a *set* is a well-defined collection of objects, *events* or outcomes. Each item within a *set* is called an *element* of the set. If every element in set 1 is also in set 2, then set 1 is a *subset* of set 2.

In a *random experiment*, each *event* or outcome is a subset of the broader *sample space*.

## sum

A *sum* is the result of adding together two or more numbers or *algebraic expressions*. In the equation  $8+6=14$ , the sum is 14.

## supplementary

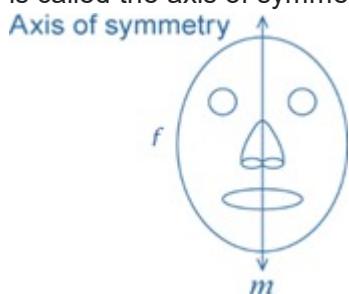
Two angles that add to  $180^\circ$  are called *supplementary angles*; for example,  $45^\circ$  and  $135^\circ$  are supplementary angles.

## surd

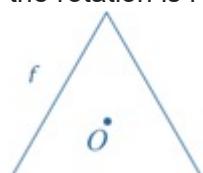
A *surd* is a numerical expression involving one or more irrational roots of numbers. Examples of surds include  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$  and  $4\sqrt{3} + 7\sqrt{3}$ .

## symmetry

A plane figure  $f$  has line symmetry in a line  $m$ , if the image of  $f$  under the reflection in  $m$  is  $f$  itself. The line  $m$  is called the axis of symmetry.



A plane figure  $f$  has rotational symmetry about a point  $O$  if there is a rotation such that the image of  $f$  under the rotation is  $f$  itself.

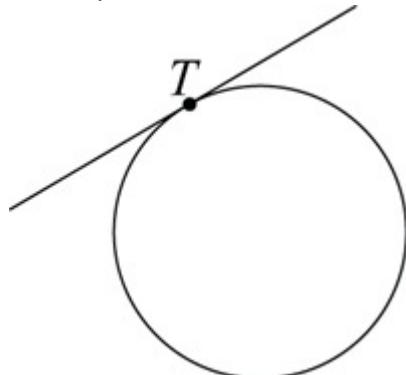


A rotation of  $120^\circ$  around  $O$  moves the equilateral triangle onto itself.

## tangent

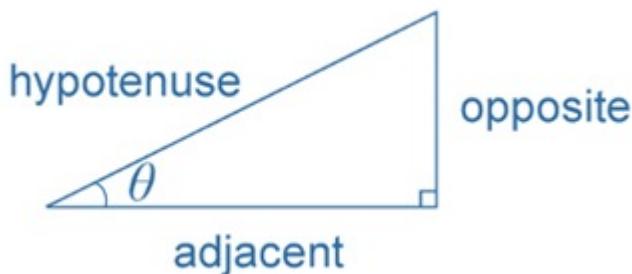
### 1. Plane-geometry:

In plane-geometry, a tangent to a circle is a line that intersects a circle at just one point. It touches the circle at that point of contact, but does not pass inside it.



### 2. Trigonometry:

In any right-angled triangle, the tangent of an angle is defined as the length of the side opposite the angle divided by the length of its adjacent side;  $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ , where  $0^\circ < \theta < 90^\circ$ .



## theorem

A *theorem* is a mathematical statement that has been established by means of a *proof*.

## three-dimensional

An object is *three-dimensional* when it possesses the dimensions of height, width and depth. *Two-dimensional* objects only have two dimensions: length and width. A *solid* is any geometrical object with three-dimensions.

## transformation

The *transformations* included in this glossary are *enlargements*, *reflections*, *rotations*, and *translations*.

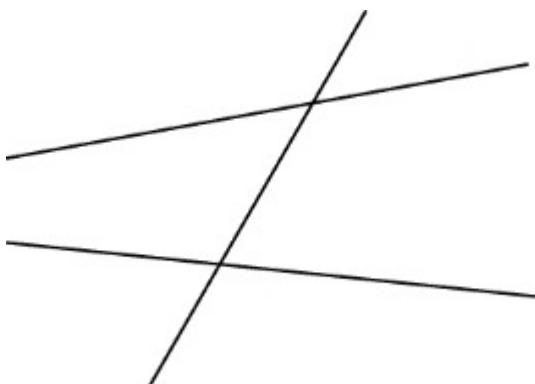
## translation

Shifting a figure in the plane without turning it is called *translation*. To describe a translation in the plane, it is enough to say how far left or right and how far up or down the figure is moved.

A translation is a *transformation* that moves each *point* to its translation image.

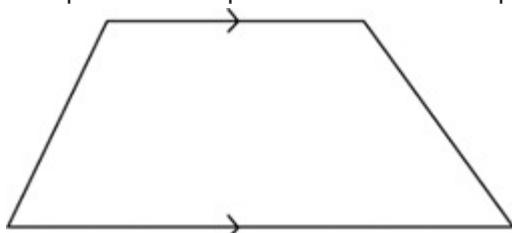
## transversal

A transversal is a line that crosses two or more other lines in a plane.



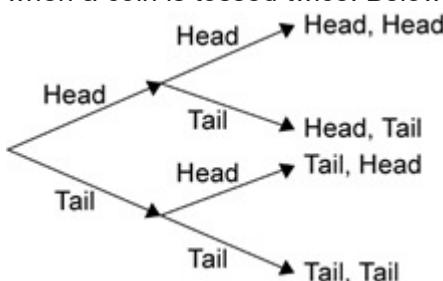
## trapezium

A trapezium is a quadrilateral with one pair of opposite sides parallel.



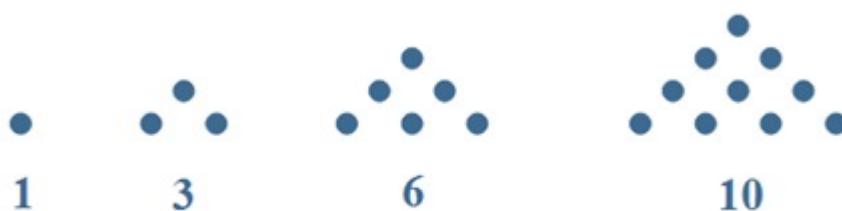
## tree diagram

A tree diagram is a diagram that can be used to enumerate the outcomes of a multi-step random experiment. The diagram below shows a tree diagram that has been used to enumerate all of the possible outcomes when a coin is tossed twice. Below is an example of a two-step random experiment.



## triangular number

A triangular number is the number of dots required to make a triangular array of dots in which the top row consists of just one dot, and each of the other rows contains one more dot than the row above it. So the first triangular number is 1, the second is  $3=1+2$ , the third is  $6=(1+2+3)$  and so on.



## trigonometric ratios

*Trigonometric ratios* describe the relationships between the *angles* and sides of right triangles. The three basic trigonometric ratios covered in this glossary are: *Sine*, *Cosine*, and *Tangent*.

## two-dimensional

A shape is *two-dimensional* when it only possesses the dimensions of length and width.

## two-way table

A two-way table is commonly used to for displaying the two-way frequency distribution that arises when a group of individuals or things are categorised according to two criteria; for example, the two-way table below displays the two-way frequency distribution that arises when 27 children are categorised according to hair type (straight or curly) and hair colour (red, brown, blonde, black).

	Curly hair	Straight hair	Total
Red hair	1	1	2
Brown hair	8	4	12
Blonde hair	1	3	4
Black hair	7	2	9
Total	17	10	27

The information in a two-way table can also be displayed graphically using a side-by-side column graph.

## unit fraction

A *unit fraction* is a simple *fraction* whose numerator is 1, that is, a fraction of the form  $1/n$ , where  $n$  is a *natural number*.

## variable

In statistics, a *variable* is something measurable or observable that is expected to either change over time or between individual observations. Examples of variables in statistics include the age of students, their hair colour or a playing field's length or its shape.

*Numerical variables* are variables whose values are numbers, and for which arithmetic processes such as adding and subtracting, or calculating an average, make sense.

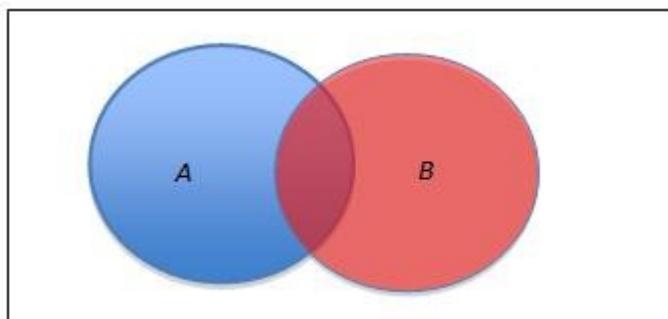
Examples include the number of children in a family or the number of days in a month.

A *discrete numerical variable* is a *numerical variable*, each of whose possible values is separated from the next by a definite 'gap'. The most common numerical variables have the counting numbers  $0, 1, 2, 3, \dots$  as possible values. Others are prices, measured in dollars and cents.

In algebra, a *variable* is a symbol, such as  $x, y$  or  $z$ , used to represent an unspecified number of a specific type; for example, the variable  $x$  could represent an unspecified *real number*.

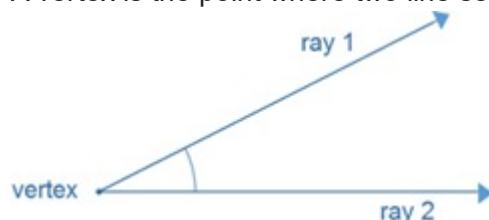
## Venn diagram

A Venn diagram is a graphical representation of the extent to which two or more events, for example A and B, are mutually inclusive (overlap) or mutually exclusive (do not overlap).



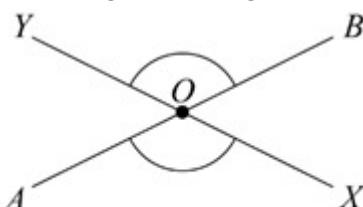
## vertex

A vertex is the point where two line segments or rays meet, join, or intersect.



## vertically opposite angle

When two lines intersect, four angles are formed at the point of intersection. In the diagram, the angles marked  $\angle AOX$  and  $\angle BOY$  are called vertically opposite. Vertically opposite angles are equal.



Vertically opposite angles are equal

## volume

The *volume* of a *solid* is a measure of the space enclosed by the solid.  
For a rectangular prism,  $Volume = Length \times Width \times Height$ .

## whole number

A *whole number* is a *non-negative integer*, that is, one of the numbers  $0, 1, 2, 3, \dots$ .  
Sometimes it is taken to mean only a *positive integer*.